Generalizing smooth transition autoregressions

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Abstract

We introduce a new model capable to parametrize the joint asymmetry in duration and length of cycles in US macroeconomic time series by using particular generalization of the logistic function. The so Generalized STAR nests the traditional symmetric and linear models as peculiar cases. A test for the null hypothesis of dynamic symmetry is discussed. Two cases studies on the index of industrial production and the unemployment rate indicate that dynamic asymmetry is a key feature of the US business cycle. Our model beats its competitors in point forecasting, while this superiority becomes less evident in density forecasting also in uncertain forecasting environments.

Keywords: Business Cycle, Density forecasts, Dynamic Asymmetry, Econometric modelling, Evaluating forecasts, Nonlinear time series, Point forecasts, Uncertainty.

JEL: C22, C51, C52

1 Introduction

The US business cycle is characterized by asymmetric fluctuations, as confirmed by a consolidated literature; see, *inter alia*, Milas et al. (2006) and therein mentioned references. Anyway, defining asymmetry is a non-trivial issue. In fact, Sichel (1993) classifies two types of asymmetry, namely (i) the *steepness*, when contractions in the levels are steeper than expansions (that is, asymmetry in the level axis) and (ii) the *deepness*, when the series undergoes at an accelerating time until a minimum after which it starts to recover with high, decreasing acceleration, until to smoothly recover the peak (that is, asymmetry in time axis). When these two definitions are combined, we say there is *dynamic asymmetry*. McQueen and Thorley (1993) use the expression *sharpness* as a consequence of the sharpness of business cycle path in recession against the more roundness in expansions.

The primary interest of this paper is in out-of-sample forecasting for the US index of industrial production (IIP) and unemployment rate (UN). We consider samples either at quarterly and at monthly frequency for each of these variables[1]. These data are displayed in Figure 1 and Figure 2. Several facts arise: (i) in both the series the dynamic asymmetry in the form of steepness is dominant during the the oil-crisis in 1975-76, 1981 and and 2001; in the fifties, sixties and 1973, and 1981-82 the cycle seems to be more characterized by deepness. (ii) The dynamic asymmetry in form of sharpness is in all its dramatic strength in the Great Recession of 2007-9: a dozen of observations are sufficient to bring the series at the levels of mid 90s while, for the recovery phase more than four times are required in the case of IIP; in the case of UN, this finding is so enforced to make the increase of the variance of the full sample evident. (iii) Different measure of growth rates conveys a different types of asymmetries. In particular, monthly growth rates amplifies the sharpness of recession periods while roundness is partially sacrificed due to a large number of short-run cycles; on the other side, yearly growth rates emphasize the deepness in

[1] All the series are in real time and can be downloaded by OECD - Main Economic Indicators.

recessions and "clean" the roundness in expansions. (iv) Anyway, no matter of the transform, the UN cycle in phase of recession has a magnitude of four/five times the IIP ones, and the strength of the cycles is augmenting since the DotCom bubble in 2001.

Hence, a good forecasting model have to incorporate the dynamic asymmetry in all its possible faces. How to do? The majority of this literature – in particular the first generation as Neftçi (1984); De Long and Summers (1984); Rothman (1991) – uses a piece-wise linear autoregression with Markov-Switching mean or variance (MSAR) with a pre-specified number of unobserved states (generally two, corresponding to the phases of Recession and Expansion); eventually, the more asymmetric is the process, the higher is the number of states in the MSAR model. This approach has been particularly appreciated for its easy implementation and close connections with algorithmic rules for dating, see Harding and Pagan (2006) for recent developments. In this paper, we adopt the alternative strategy to treat the process as a continuum of observable states oscillating between two extremes and fit a general, flexible nonlinear function over the observables. This is possible by using smooth transition autoregression (STAR) models introduced by (Haggan and Ozaki, 1981; Chan and Tong, 1986) and developed by Teräsvirta (1994). These are represented by a sum of a linear part and a nonlinear autoregressive part constituted by a function of the transition variable and a location parameter. In particular, a logistic transition is commonly postulated when the series under consideration is assumed having asymmetric oscillations from its conditional mean.

We argue that, being the logistic function reflectively symmetric by construction, the resulting logistic STAR does not match the theoretical definition of dynamic asymmetry. In other words, the available models for time series allow the econometrician, at the best, to answer to the question: *Does the series return to its original regime and when?* Here, our objective is to answer to another, more challenging question: *Does the rate of change (if any) in the left tail of the logistic transition* *differ from the right tail and how much?* As we will show, an appropriate solution to this methodological question, *per se* interesting for descriptive aims, improves the forecasting ability of STAR models family.

Two different solutions are nowadays available: the first, proposed by Sollis et al. (1999) (SLN1) and Lundbergh and Teräsvirta (2006) (LT) is to raise the STAR's transition function to an exponent; the second, suggested by Sollis et al. (2002) (SLN2) is to add a parameter inside the transition function in such a way to control for the asymmetry of both the tails of the transition function by simply using a Heaviside indicator. Unfortunately, both of these solutions present some criticality: Figure 3, panel (a) clearly shows that in the SLN2 case, the transition function could be non-smooth; on the other hand, the SLN1 and LT parametrization, plotted in panel (b) conveys a smooth transition, but the effect of increasing of the asymmetry parameter could translate just in a shift effect, if not properly restricted as stated in the same article; moreover, this parametrization does not provide an immediate description of the behaviour of each tail of the transition function (which is instead the beauty of SLN2). Thus, the detection and assessment of the dynamic asymmetry in a statistically well-specified time series model seem still an open issue.

In the next Section 2 we contribute to this strand of literature by applying to the autoregressive (AR) model a generalized version of the logistic transition function with two parameters governing the two tails of the logistic sigmoid and a logarithmic/exponential rescaling able to preserve the smoothness of the transition. The resulting Generalized STAR (GSTAR) model encloses the symmetric STAR – and thus, AR – as special cases. An LM-type test for the null hypothesis that the two tails of the transition function are reflexively symmetric is discussed in Section 3. Section 4, the forecasting properties of the GSTAR model for the U.S. data, jointly with a discussion on the relevance of the empirical finding. Finally, Section 5 summarizes and concludes. A Supplement provides details on alternative asymmetry test and diagnostics, simulations, mathematical derivations and additional examples.

2 Forecasting Models

This section provides a brief description of the non-linear and dynamic asymmetric models implemented in our analysis. For details in statistical inference, model specification and parameter estimation we refer readers to Teräsvirta et al. (2010), and to Canepa and Zanetti Chini (2016) for what concerns dynamic asymmetric specification. In what follows, we adopt "$\doteq$" to mean "equal by definition" and "$\equiv$" to indicate an equivalence; bold is used for matrix notation; y_t is a realization of a (univariate) time series observed at $t = 1-p, 1-(p-1), \ldots, -1, 0, 1, \ldots, T-1, T$. All of the estimated models for quarterly (monthly) samples include four (eight) lags; however, all the below written formulas refer to general autoregressive order p. Finally, in our application all the transition variables are assume to be a lag of the dependent variable; thus, none of the models treated in this section requires exogenous variables.

2.1 GSTAR Models

The process $\{y_t\}_t^T$ follows a GSTAR(p) model if

$$y_t = \boldsymbol{\phi}'\mathbf{z}_t + \boldsymbol{\theta}'\mathbf{z}_t G(\boldsymbol{\xi}) + \epsilon_t, \qquad \epsilon_t \sim I.I.D.(0, \sigma^2), \tag{1}$$

$$G(\boldsymbol{\xi}) = \left(1 + \exp\left\{-\prod_{k=1}^{K} h\big(c_k, s_t\big)\right\}\right)^{-1}, \tag{2}$$

where the $T \times 1$ vector y_t is a dependent variable, $\mathbf{z}_t = (1, y_{t-1}, \ldots, y_{t-p})'$, $\boldsymbol{\phi} = (\phi_0, \phi_1, \ldots, \phi_p)'$, $\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_p)'$ are autoregressive parameter vectors, $G(\boldsymbol{\xi}) \doteq G(\boldsymbol{\gamma}, h(c_k, s_t))$ is a transition function of the vector of nonlinear parameters $\boldsymbol{\xi} = [\boldsymbol{\gamma}, h(c_k, s_t)]$, which in turn is formed by the vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2))$ and a function of the K location parameter c_k, the transition variable $s_t = y_{t-d}$, with $d > 0$ denoting the delay, and defining $\eta_t \equiv (s_t - c)$ for ease of notation,

$$h(\eta_t) \doteq \begin{cases} \gamma_1^{-1} \exp(\gamma_1 |\eta_t| - 1) & \text{if } \gamma_1 > 0, \\ 0 & \text{if } \gamma_1 = 0, \\ -\gamma_1^{-1} \log(1 - \gamma_1 |\eta_t|) & \text{if } \gamma_1 < 0, \end{cases} \tag{3}$$

for $\eta_t \geq 0$ ($\mu > 1/2$) and

$$h(\eta_t) \doteq \begin{cases} -\gamma_2^{-1} \exp(\gamma_2 |\eta_t| - 1) & \text{if } \gamma_2 > 0, \\ 0 & \text{if } \gamma_2 = 0, \\ \gamma_2^{-1} \log(1 - \gamma_2 |\eta_t|) & \text{if } \gamma_2 < 0, \end{cases} \tag{4}$$

for $\eta_t < 0$ ($\mu < 1/2$). The transition function $G(\cdot, \cdot)$ is continuous in $\boldsymbol{\gamma}$. Equation (3) (equation (4)) models the higher (lower) tail of the probability function, so allowing for the asymmetric behaviour introduced by the slope parameter γ_1 (γ_2) which controls the velocity of the transition. When $\gamma_1, \gamma_2 > 0$ ($\gamma_1, \gamma_2 < 0$), $h(\eta_t)$ is an exponential (logarithmic) rescaling which increases more quickly (more slowly) than a standard logistic function. This parametrization in $h(\cdot)$ is necessary in order to build a test for the null of linearity against of dynamic asymmetry but is not the only possible. In particular, (1)–(4) assumes $K = 1$ and can be generalized to other distributions of exponential family[2]. The Generalized Logistic is plotted in Figure 3: the resulting sigmoid is clearly consistent with the Sichel (1993) definition of dynamic asymmetry (see, e.g., the case in which $\gamma_1 = -2$ and $\gamma_2 = 4$) and maintains the global slope of the transition function unchanged with respect to the traditional symmetric model. notice the straight line aroung 0.5 which corresponds to linear AR model in the case of $\boldsymbol{\gamma} = \mathbf{0}$. There a are three particular cases: first, when $h(\eta_t) = \eta_t$ implies that the function nests a one-parameter symmetric logistic

[2]Another other important case is $K = 2$, corresponding to the generalized exponential STAR (GESTAR), where parameters $\boldsymbol{\phi} + \boldsymbol{\theta} G(\gamma, \boldsymbol{c}, s_t)$ change asymmetrically around the mid-point $(c_1 + c_2)/2$, where the generalized logistic function attains its minimum, $min_G G(\cdot) \in [0, 1/2]$. This specification, however is not used in our application and is not discussed; see Canepa and Zanetti Chini (2016) and Supplement for details.

STAR model with slope $\gamma_1 = \gamma_2 = \gamma$. Second, when $\boldsymbol{\gamma} \to +\infty$, $G(\cdot,\cdot)$ nest an indicator function $I_{(s_t>c)}$. Third, when $\boldsymbol{\gamma} = 0$, $G(\cdot,\cdot)$ nest a straight line around $1/2$ for each s_t. Each case is described in the course of this section.

An important assumption is that $Q(z) = z^p - \phi_1 z^{p-1} - \cdots - \phi_p = 0$ has its roots inside the unit circle if the process is characterized by $G(\mathbf{0}, h(\eta_t))$; this implies that the model is stationary and ergodic under the null hypothesis of linearity[3]. Finally, $\{\epsilon_t\}_t^T$ is assumed to be a martingale difference sequence with respect to the history of the time series up to time $t-1$, denoted as $\Omega_{t-1} = [y_{t-1}, \ldots, y_{t-p}]$, i.e., $E[\epsilon_t|\Omega_{t-1}] = 0$. This is sufficient to built up tests based on artificial regressions as demonstrated in Davidson and McKinnon (1990) and has important consequence for applied aims, in what the test discussed in Section 3 and the three diagnostic tests discussed in Supplement can still be meaningful if the normality hypothesis is rejected.

We remark that the above described GSTAR is the time series variant of the original generalized logistic model proposed by Stukel (1988), which differs for the definition of $h(\cdots)$ in the case of $\gamma_1 = 0$ and $\gamma_2 = 0$, $\mathbf{z} = (x_1, \ldots, x_N)'$, for $\{x_i\}_{i=1}^N$ being N exogenous regressors, and consequently, $\eta_t = \boldsymbol{\phi}'\mathbf{z}$.

2.2 STAR Models

When $\gamma_1 = \gamma_2$ in (1)–(4), the GSTAR model nests a traditional (possibly, Multiple Regime)-STAR model (MRSTAR, henceforth):

$$y_t = \boldsymbol{\phi}'\mathbf{z_t} + \boldsymbol{\theta}'\mathbf{z_t} \sum_{m=1}^{M} G(\boldsymbol{\gamma}, \mathbf{c}, s_t) + \epsilon_t, \quad \epsilon_t \sim i.i.d.(0, \sigma^2), \tag{5}$$

where y_t is the dependent variable, $\mathbf{z_t} = (1, y_1, \ldots, y_{t-p})'$, $\boldsymbol{\phi} = (\phi_0, \phi_1, \ldots, \phi_p)'$, $\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_p)'$ are parameter vectors. The transition function $G(\boldsymbol{\gamma}, \boldsymbol{c}, s_t)$ is a continuous function in the transition variable s_t, where the parameter vector $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_m, \ldots \gamma_M)$ controls the velocity of the M transitions with $\mathbf{c} =$

[3]This assumption can be relaxed, as in Kapetanios et al. (2003); Vougas (2006).

$(c_1, \ldots, c_m, \ldots c_M)$ assumed as a vector of transition parameters. In what follows we suppose that the transition variable coincides with a lagged value of the endogenous variable y_t with lag denoted by delay $d > 0$.

When $K = 1$, equations (6) and (5) define the first-order (Multiple-Regime) Logistic STAR (MRLSTAR) model,

$$G(\boldsymbol{\gamma}, \boldsymbol{c}, s_t) = \left(1 + exp\left\{-\gamma_M \prod_{k=1}^{K}(s_t - c_m)\right\}\right)^{-1}, \tag{6}$$

$$\gamma_1 > 0, \ldots, \gamma_m > 0, \ldots, \gamma_M > 0, \quad c_1 < \cdots < c_m < \cdots < c_M \tag{7}$$

where conditions on γ and c in equation (7) are identifying restrictions. This make the symmetric (MR)STAR fundamentally different to GSTAR in (1), where no additional identification restriction is needed. Under logistic specification, $\boldsymbol{\phi} + \boldsymbol{\theta} G(\gamma, \boldsymbol{c}, s_t)$ change monotonically as a function of s_t from $\boldsymbol{\phi}$ to $\boldsymbol{\phi} + \boldsymbol{\theta}$. The STAR model is capable to produce accurate forecasts of the process and allows to mimic the business cycle, see Anderson and Teräsvirta (1992); Rothman (1998), *inter alia.*

2.3 (SE)TAR Models

When $\gamma_1 = \gamma_2 \to \infty$ in (1)–(4) (or, equivalently $\gamma_m \to \infty$ the model (5), the (G)STAR nests a SETAR model (Tong, 1983):

$$y_t = \sum_{j=1}^{r+1} (\boldsymbol{\phi}_j' \boldsymbol{z_t}) I\big(y_{t-d} \leq c_j\big) + \sum_{j=1}^{r+1} (\boldsymbol{\phi}_j' \boldsymbol{z_t}) I\big(y_{t-d} > c_j\big) + \epsilon_{jt} \quad \epsilon_{jt} \sim i.i.d.(0, \sigma_j^2) \tag{8}$$

where $\boldsymbol{\phi}, \boldsymbol{z_t}$ are defined as in previous models, s_t is a continuous switching random variable, $c_0, c_1, \ldots, c_{r+1}$ are threshold parameters, $c_0 = -\infty$, $c_{r+1} = +\infty$, $j = 1, \ldots, r$. The multiple regime hypothesis is investigated via LM test, and the most likely number of regimes can be obtained by iteration.

2.4 AR Models

The linear Auto-regressive model (AR)

$$y_t = \boldsymbol{\phi}'\mathbf{z_t} + \epsilon_t, \quad \epsilon_t \sim iid(0, \sigma^2), \tag{9}$$

where $\mathbf{z}_t = (1, y_{t-1}, \ldots, y_{t-p})'$, and $\boldsymbol{\phi} = (\phi_0, \phi_1, \ldots, \phi_p)'$, is a peculiar case of model (1)–(4) with $\gamma_1 = \gamma_2 = 0$ (or equivalently, of model (5) with $\gamma = 0$) and $M = 1$. In a such situation, the transition function is $G(\gamma_m, \boldsymbol{c}, s_t) \equiv 1/2$, exactly as shown in Figure 3.

3 Testing for Dynamic Symmetry

Consider (1) with $G(\boldsymbol{\gamma}, h(\eta_t))|_{\boldsymbol{\gamma}=\mathbf{0}}$ and define $\boldsymbol{\tau} = (\boldsymbol{\tau_1}, \boldsymbol{\tau_2})'$, where $\boldsymbol{\tau_1} = (\phi_0, \boldsymbol{\phi}')'$, $\boldsymbol{\tau_2} = \boldsymbol{\gamma}$. Let $\hat{\boldsymbol{\tau}}_\mathbf{1}$ the LS estimator of $\boldsymbol{\tau_1}$ under $H_0 : \boldsymbol{\gamma} = \mathbf{0}$, $\hat{\boldsymbol{\tau}} = (\boldsymbol{\tau}_\mathbf{1}', \mathbf{0}')'$. Moreover, let $\mathbf{z_t}(\boldsymbol{\tau}) = \frac{\partial \epsilon_t}{\partial \boldsymbol{\tau}}$ and $\hat{\mathbf{z}}_\mathbf{t} = \mathbf{z_t}(\hat{\boldsymbol{\tau}}) = (\hat{\mathbf{z}}_{\mathbf{1},\mathbf{t}}, \hat{\mathbf{z}}_{\mathbf{2},\mathbf{t}})$, where the partition conforms to that of $\boldsymbol{\tau}$. Then the general form of LM statistic is:

$$S(\Xi)^{LM} = \frac{1}{\hat{\sigma}^2}\hat{\mathbf{u}}'\hat{\mathbf{Z}}_\mathbf{2}(\hat{\mathbf{Z}}_\mathbf{2}'\hat{\mathbf{Z}}_\mathbf{2} - \hat{\mathbf{Z}}_\mathbf{2}'\hat{\mathbf{Z}}_\mathbf{1}(\hat{\mathbf{Z}}_\mathbf{1}'\hat{\mathbf{Z}}_\mathbf{1})^{-1}\hat{\mathbf{Z}}_\mathbf{1}'\hat{\mathbf{Z}}_\mathbf{2})^{-1}\hat{\mathbf{Z}}_\mathbf{2}'\hat{\mathbf{u}}, \tag{10}$$

where $\hat{\mathbf{u}}$ is previously defined, $\hat{\sigma}^2 = \frac{1}{T}\sum_1^T \hat{u}_t^2$ and $\hat{u}_t = y_t - \hat{\boldsymbol{\tau}}_\mathbf{1}'\mathbf{z_t}$, $\hat{\mathbf{Z}}_\mathbf{i} = (\hat{\mathbf{z}}_\mathbf{i1}, \ldots, \hat{\mathbf{z}}_\mathbf{it}, \ldots, \hat{\mathbf{z}}_\mathbf{iT})'$, $i = \{1, 2\}$, $t = 1, \ldots, T$. When the model is an GLSTAR, $\hat{\mathbf{z}}_{\mathbf{1},\mathbf{t}} = -\mathbf{z_t} = -(1, y_{t-1}, \ldots, y_{t-p})'$ while $\hat{\mathbf{z}}_\mathbf{2t} \equiv \frac{\partial^2 u_t}{\partial\gamma\partial\gamma'}\big|_{\boldsymbol{\gamma}=\mathbf{0}} = -\frac{1}{2}\{\theta_{20}[y_t(y_{t-d})] - cy_t\boldsymbol{\theta}'\mathbf{z_t} + \boldsymbol{\theta}_\mathbf{2}'\mathbf{z_t}y_t y_{t-d}\}$, where d is the delay parameter. The change in the definition of $\mathbf{z_{2t}}$ is not significant in terms of LM statistic build-up. This implies that no change of treatment with respect to the original parametrization is needed. In particular, in order to circumvent the Davies (1977)' s problem of unidentification of nuisance parameters θ_0 and $\bar{\boldsymbol{\theta}} = [\theta_1, \ldots, \theta_p]'$ under the null hypothesis, the same LST approach can be used. The linearized GLSTAR model leads to the following auxiliary regression for testing linearity and symmetry:

$$\hat{u}_t = \hat{\mathbf{z}}'_{1t}\tilde{\boldsymbol{\beta}}_1 + \sum_{j=1}^{p}\beta_{2j}y_{t-j}y_{t-d} + \sum_{j=1}^{p}\beta_{3j}y_{t-j}y_{t-d}^2 + \sum_{j=1}^{p}\beta_{4j}y_{t-j}y_{t-d}^3 + v_t \;, \tag{11}$$

where v_t is a $N.I.D.(0,\sigma^2)$ process, $\tilde{\boldsymbol{\beta}}_1 = (\beta_{10}, \boldsymbol{\beta}'_1)'$, $\beta_{10} = \phi_0 - (c/4)\theta_0$, $\boldsymbol{\beta}_1 = \boldsymbol{\phi} - (c/4)\boldsymbol{\theta} + (1/4)\theta_0\mathbf{e}_d$, $\mathbf{e}_d = (0, 0, \ldots, 0, 1, 0, \ldots, 0)'$ with the d-th element equal to unit and $T_3(G) = f_1 G + f_3 G^3$ is the third-order Taylor expansion of $G(\Xi)$ at $\boldsymbol{\gamma} = \mathbf{0}$, $f_1 = \partial G(\Xi)/\partial\Xi\big|_{\boldsymbol{\gamma}=\mathbf{0}}$ and $f_3 = (1/6)\partial^3 G(\Xi)/\partial\Xi\big|_{\boldsymbol{\gamma}=\mathbf{0}}$, $G(\Xi)$ being defined in previous section. The null hypothesis is

$$H_0 : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0 \;\; j = 1, \ldots, p, \tag{12}$$

The test statistic:

$$LM_1 = (SSR_0 - SSR)/\hat{\sigma_v}^2 \;, \tag{13}$$

with SSR_0 and SSR denoting the sum of squared estimated residuals from the estimated auxiliary regression (11) and under the null and alternative, respectively and $\sigma_v^2 = (1/T)SSR$, has an asymptotic χ^2_{3p} distribution under H_0. Similar argument, with different definitions of $\hat{u}_t$, $\tilde{\boldsymbol{\beta}}_1$, $\boldsymbol{\beta}_1$, H_0, holds for the exponential and double exponential cases; see Canepa and Zanetti Chini (2016).

4 Illustrations

4.1 Set-up

In this section the GSTAR model is applied to the US data introduced in previous Section 1. The peculiar logarithmic/exponential rescaling property of the generalized logistic transition function makes our parametrization particularly useful to fit the variables in growth rates. In order to control for the possible dominance of deepness against sharpness while considering (infra-)yearly growth rates, we repeat our investigation for both yearly and infra-yearly growth rates. Thus we estimate eight different models, labeled as $q\Delta^q \log(\cdot)$ with $q = \{1; 4\}$ denoting the quarterly/yearly difference when using sample with quarterly frequency and $m\Delta^m \log(\cdot)$ with $m = \{1; 12\}$ the equivalent measure when the frequency is monthly[4].

The literature point forecasting and on evaluation of individual density forecasts under linear models is nowadays established, see Timmermann (2006) and Corradi and Swanson (2006). We adopt four different measures of point forecast accuracy and four other measures for density forecast accuracy[5].

When the model is nonlinear, the one-step forecast is immediately available if knowing the nonlinear function in what, by least-square criterion, $E(\epsilon_{t+1}|I_t) = 0$, $I_t = y_{t-i}, i \geq 1$ in (1). The multi-step ahead forecast is not available in closed form and requires numerical integration. Hence at t+1, we generate, $1, \ldots, m, \ldots, M$ draws, for example, from model (1) – (4) conditionally on the estimated nonlinear parameters $\boldsymbol{\xi}$ and obtain the forecast $y_{t+1} \sim f(y_{t+1} + \epsilon_{t+1}^{(m)}; \boldsymbol{\xi}|I_t)$, which is called skeleton forecast of y_t; in turn, this is collected to draw, at $t+2$, the forecast $y_{t+2} \sim f(y_{t+2} + \epsilon_{t+2}^{(m)}; \boldsymbol{\xi}|I_t, y_{t+1}^{(m)})$, and so on until, at $t+h$,the forecast $y_{t+h|t} = f(y_{t+h} + \epsilon_{t+h}|I_t, y_{t+1}^{(m)}, \ldots, y_{t+h-1}^{(m)})$ is obtained and then evaluated as:

$$y_{t+h}^{MC} = \frac{1}{M} \sum_{m=1}^{M} y_{t+h|t}^{(m)}. \tag{14}$$

The Monte-Carlo approach requires to make assumptions on the distribution of random numbers ϵ_t. As we will see in the course of the section, assuming a distribution in random number generation has severe implications in our examples, in particular when density forecasting is required. This problem can be partially avoided by block-bootstrapping the original sample series. Namely, the series is divided in blocks of magnitude $b > 1$, which then are sampled with replacement and this for

[4]For easy of notation, the "1" up to Δ is deleted.

[5]These measures are just a fraction of the many nowadays available. Their choice has been done for easy of treatment and ease of comparison in literature, *ergo* does not imply any preference.

every possible contiguous element in the original sample; thus the sampled blocks are attached obtaining the new bootstrap series $(\tilde{y}_t^{(1)}, \ldots, \tilde{y}_t^{(i)}, \ldots, \tilde{y}_t^{(B)})$ from the same model; finally, we sequentially compute the M_B^b forecasts for $\tilde{y}_{t+1}, \tilde{y}_{t+2}, \ldots, \tilde{y}_{t+h}$ as before described up to arrive at the skeleton $\tilde{y}_{t+h} = g(\tilde{y}_{t+h} + \tilde{\epsilon}_{t+h}^{(i)} | I_{t+h-1})$ and evaluating:

$$\tilde{y}_{t+h}^B = \frac{1}{M_B^b} \sum_{B=1}^{M_B^b} \tilde{y}_{t+h|t}^{(B)}. \tag{15}$$

In our application we adopt a moving block-bootstrap algorithm with $b = 10$ and $B = 10,000$ draws. This allows to avoid to make assumptions on the distribution of estimated residuals, and thus to have a forecast robust to model parameter uncertainty, see Efron and Tibshirani (1993), CH. 8.

Once the forecasts from different models, it is interesting to ask if our model performs better than their linear and symetric competitor(s). This is done by using the Diebold and Mariano (1995)[6], the Giacomini and White (2006) and Amisano and Giacomini (2007) tests for equal predictive ability in order to compare couples of forecasts. Under the null hypothesis, there is no evidence of superiority of the GSTAR on its linear or symmetric equivalent.

4.2 The U.S. Industrial Production

According to the p-values reported in Table 1, the null hypotheses of linearity and dynamic symmetry are always rejected if considering the significance threshold at 0.10. Nevertheless, detecting nonlinearity – and, *a fortiori*, dynamic asymmetry – is more difficult when infra-yearly growth rates are considered. Just to make an example, in the quarterly series in quarterly growth rates ($q\Delta logIIP$) the $p-$values are relatively high (0.0662 and 0.0447) if compared to the ones corresponding to the

[6] Since the AR(p) and STAR(p) models are nested in GSTAR(p) specification here proposed, the inference of this test is severely biased, as proved in West (1996) and other equivalent tests as the one by Clark and McCracken (2001) should be employed. Nevertheless, we will still use the classical Diebold-Mariano for a matter of preliminary check. As we will see in our illustrations, the resulting p-values are often counterintuitive. In any case, the p-values of Clark-McCraken test do not change our conclusions and thus they are omitted for a matter of space.

series in yearly growth rates ($q\Delta^4 logIIP$, which $p-$values is 0.0037 for the null of linearity and 0.0005 for the null of symmetry). Monthly data samples confirm such a weak asymmetry.

None of the models require the use of a second transition function in order to capture additive dynamic asymmetry. In a similar fashion, there is no evidence of time-varying parameters. Diagnostic tests for the null of serial uncorrelation are often lower than 0.05. Nevertheless, the effects in terms of $p-$value are limited although by increasing p considerably; thus, being the other diagnostic tests passed, the choice of $p = 4$ looks merely to the parsimony.

In three series (both quarterly samples and monthly sample in yearly growth rates) this the estimated slopes have opposite signs (γ_1 negative and γ_2 positive, and in most the case significative; their magnitude is higher in γ_1. On the other hand, in $m\Delta logIIP$ both of them are positive; not surprisingly, this is the case where $p-$values of linearity and asymmetry tests are higher. The estimated transition functions of GSTAR model and their (MR)STAR equivalent for samples in quarterly frequency are plotted in Figure 4. The GSTAR transitions differ from their symmetric equivalent[7]. In particular, in panels (a) where $q\Delta^4 logIIP$ is displayed, the estimated $\hat{G}$ tends to concentrate in the upper part of the of the space of continuum of states (around 0.7) and just few observations are above 0.5, generally, the ones corresponding to NBER recessions. This sounds as confirmation of deepness as dominant feature. The symmetric STAR behaves as a step function with almost all observation taking value 1 and some 0 – more similar to SETAR model. On the other side, in $q\Delta^4 logIIP$ displayed in in panels (b) of the same figure, STAR transition is almost entirely near 0.5 – indicating, in a such a way, an almost linear behavior; instead, GSTAR is in the range [0, 0.9] with prevalence of [0, 0.5] after first 1980s. This means that STAR models cannot capture dynamic asymmetry, at least in this example. The transition functions corresponding to series in monthly

[7] The parameter estimates of (MR)STAR are not shown for space reasons and are available under request.

frequency are shown Figure 5: again, the GSTAR model reproduces the deepness in $m\Delta^{12}logIIP$. In fact, while the observations in state 0 coincides with STAR ones, the majority of the others are never upper 0.7. Indeed, the slope function is highly asymmetric so that sigmoid does never reach 1. The situation for $m\Delta logIIP$ is almost specular: the majority of the observation are almost uniformly in the range [0.5, 0.9] with recessions characterized by few observations near the state 0; this produces a slope near to a pure logistic sigmoid. Notice that models for monthly data requires at least two transition functions in the symmetric specification.

Table 2 reports the predictive performances of GSTAR for quarterly data jointly with the results for a linear autoregression and the symmetric STAR models. Our parametrization is significantly preferable to both the alternatives for three measures of point forecasting and two density measures on a total of four measures considered. Interestingly, if considering LogS, the linear specification in the short-run while nonlinear symmetric model is preferred for medium term forecasts (2- and 4-ahead quarters). The forecasting performances of monthly series are reported in Table 3. Almost specularly to what previously written for quarterly models, the GSTAR model is preferred in three point measures on four and just one density measure (the LogS) on four, while the quadratic and quantile scores (QS and qS, respectively) support the symmetric nonlinear parametrization; the linear model wins in CRPS.

Table 4 reports the predictive accuracy measures for the same experiment where forecasts of models for quarterly data samples are obtained by the block-bootstrap algorithm. Introducing uncertainty in the forecasting process reduces the point predictive performances of asymmetric model. The symmetric MRSTAR beats GSTAR in one measure (mRAE) and the linear autoregression is preferred in the majority of horizons in MFE and RMFE. On the other side, if density measures are considered, the asymmetric specification wins in three cases on four – that is one more the experiment with no uncertainty. The same measures under uncertainty for monthly data are reported in Table 5. In this case, GSTAR model wins for almost all leads of MFE and mRAE and the others the symmetric model prevails in other point measures. Again, GSTAR model do not prevail in density measures, being preferred only under QS.

The hypothesis of no equal predictive ability is investigated in Table 6 for models of quarterly samples and Table 7 for models of monthly data samples. The Diebold-Mariano test is not able to reject the null hypothesis of no improvement in forecasting ability for a nonlinear and dynamically asymmetric specifications with respect to linear (and symmetric) ones. This result appears counterintuitive, since the linearity and asymmetry tests suggest the converse. But since the Dyabold-Mariano statistics is though for non-nested models, we consider it as a preliminary control. In facts, the (more general) Giacomini-White test does allows for improvements in forecasting ability of GSTAR model in short-run horizons, albeit the $p-$values blows up as horizons increase and thus the evidence of an improvement decays rapidly in long-run. The Amisano-Giacomini test supports the GSTAR model only under CRPS, while in the choice between nonlinear symmetric model and linear specification, the first one is supported also by the quantile measure. In monthly samples the gain in forecasting ability of asymmetric models is considerably higher for both asymmetric and nonlinear symmetric specifications; the Amisano-Giacomini test supports nonlinear specification and, again, restricts the preference for GSTAR model to LogS.

4.3 The US. Unemployment Rate

According to the graphical inspection in Section 1, UN is strongly countercyclical and, with respect to IIP, the phases of the business cycle are exacerbated. This finding is confirmed by Table 1: linearity and the dynamic symmetry are always and uniformly rejected. In facts, the p-values of all the samples are always below 0.05, albeit the $m\Delta logUN$ with its 0.0416 is near to this limit – quite high due to the high number of observations. [8].

[8]However, in a previous version of this paper, we consider also the null hypothesis that the model is a GSTAR with different slopes, which strictly follows the original Stukel's methodology.

Similarly to IIP, all the models for UN pass the diagnostic tests for the null of no additive asymmetry and parameter constancy. The hypothesis of serial uncorrelation is rejected only in quarterly samples. Still, in model specification, we maintain four lags because no evidence of satisfactory improvement in p-values of test for serial uncorrelation unless augmenting p up to 8[9].

Coherently with the counter-cyclical nature of this indicator, the estimated slopes are opposite in sign with respect on IIP in all the series: γ_1 positive and γ_2 negative; the magnitude is higher in γ_1 with only exception of $q\Delta^4 logUN$, where these parmeters are 1 and -1.

The GSTAR transitions of $q\Delta^4 logUN$ and $q\Delta logUN$ are displayed in Figure 6. Both of them concentrate in the lower part of the of the space of continuum of states (around 0.1 and 0.3, respectively); just few observations are above 0.5, and when this happen, these go directly to 1. Indeed, the corresponding sigmoid are nicely steep and deep but with opposite to the ones of quarterly IIP: round and smooth in the lower part, abrupt in the upper one. Figure 7 displays the transition functions corresponding to series in monthly frequency and confirms the previous findings for quarterly series.

According to the comparison of predictive accuracy in Table 2, the GSTAR parametrization is significantly preferable to both AR and STAR models for three measures of point forecasting and almost all leads of all density measures. Such a supremacy of the dynamically asymmetric specification is confirmed by monthly series (Table 3) for point measures, with some exception for short-run and very long-run leads. On the opposite side, in density measures the better performance of GSTAR is an exception (namely, in the long-run horizons): while the QS supports the symmetric

This changes all the results, indicating a nonlinear asymmetric behavior only in $q\Delta^4 logUN$ and $m\Delta^1 2 logUN$series; nevertheless the evidence of Monte Carlo study for this, more restrictive tests, lead us to discourage its use, because of the huge conservativeness. See the Supplement for further details.

[9]In previous versions of this paper the UN was analyzed only in monthly frequency, and in that occasion, we set $p = 4$; an error in code of the estimation step led us to reconsider the whole modelling procedure and the diagnostic test as a consequence; we thanks an anonymous referee for observing this point.

nonlinear parametrization, in CRPS and qS the linear model is preferred.

Uncertainty annihilates all predictive performances of asymmetric model. According to Table 4, the MRSTAR beats GSTAR in two point measures (MFE and mRAE) and two density measures (QS and qS) and the linear autoregression wins in other cases. Between models for monthly samples (Table 5) GSTAR wins for almost two leads of MFE and almost all leads of mRAE and in quadratic measure for density. As for the case of quarterly IIP, the Diebold-Mariano test in Table 6 does not reject the null hypothesis of no improvement in forecasting ability for a nonlinear and dynamically asymmetric specifications. Differently, the Giacomini-White test does allows for improvements in forecasting ability of STAR model against the linear model in three horizons on four; the GSTAR model wins in short-run and medium-run horizons; when the horizons increase the $p-$values blows up and the evidence of forecasting improvement decays rapidly. The results for Amisano-Giacomini test are equivalent to the ones for IIP: the GSTAR model beats MRSTAR only under CRPS, while the MRSTAR beats AR in CRPS and qS. In models for monthly data (Table 7), both Diebold Mariano and Giacomini-White reject the null hypothesis considerably more strongly than in quarterly data; the Amisano-Giacomini test allows for improvements in forecasting ability for the case of MRSTAR models against linear only under QS. Instead, in the same test, the better GSTAR performances enlarge to long-run horizons of LogS and qS and long-run of CRPS; the MRSTAR prevails in short-run forecasts of almost all measures.

4.4 Discussion

What we learn from the above empirical investigation? Several considerations can be done. *Prima facie*, the GSTAR model characterizes the dynamic asymmetry of the US business cycle more accurately than its nonlinear competitor because cyclical movements in the data and their phases are reproduced considerably more precisely than the traditional parametrization. The two slopes parameters differs in signs and, significantly – in some case, up to 10 times. The transition functions are consistent with NBER recession dates in the majority of the cases. The effect of the new model on dating algorithm is still unknown, and the literature is almost completely focused on Markov-Switching and Threshold autoregressions, see Engel et al. (2005); Chauvet and Piger (2008).

In secundis, such a superior descriptive accuracy is generally associated to an improvement in point forecasting ability. This finding is not trivial. In fact, the forecasting ability of nonlinear models has been recently checked by Ferrara et al. (2015). These authors consider more families of models than us (MSARs and time-varying ARs) in their empirical exercise and a larger dataset and concludes that predictive gain as arising from non-linear models is not systematic and when existing, it is small[10]. They explains this evidence by Stock and Watson (2012) sequence of unusually large shocks hypothesis and concludes that using exogenous variables in time series models is globally preferable. Despite the differences in the number of models and in the dataset, our evidence confirms their conclusions only when observing that the gain in terms of forecasting performances of dynamic asymmetric models is low (albeit existent). But this holds almost exclusively when considering density measures. In particular, we confirm the evidence by Kascha and Ravazzolo (2010) that the relation between highest LogS and lower RMSFE is not one-to-one, as instead commonly (and implicitly) though[11]. Differently to Ferrara et al., however, we observe that the Great Recession provides a important motivation for the use of dynamic asymmetric models, and, in general, for the STAR family: in fact, controlling for the inclusion of data after 2007, leads to an important increase in the

[10]"*Indeed, the results are rather mixed and depend strongly on the evaluation period. However, predictive gains that stem from nonlinear models may arise from variables that experienced clear regime changes over the sample, such as interest rates, for instance. When comparing the performances of non-linear models, the TVAR model seems to be very similar to the AR model [$\cdots$] and the MSAR model often leads to the poorest results. On the other hand, the TAR and LSTAR models occasionally perform quite well*" (Ferrara et al., 2015, page 678).

[11]We underline that this result is confirmed by our additive examples in Supplement, where other (non only economic) datasets has been analyzed.

p-value of the test for dynamic asymmetry[12].

Third, we have to consider an important caveat: in our forecasting exercise, we assume $E[\epsilon_t^2|\Omega_{t-1}] = \sigma^2$, that is that the conditional variance of the process $\{\epsilon_t\}_t^T$ is constant. According to Clark and Ravazzolo (2015), incorporating stochastic (time-varying) volatility in simple macroeconometric models improves substantially their forecasting properties. Recent advancements in this strand of literature allow to relax this restriction in STAR models, see Gonzáles-Rivera (1998); Lundbergh et al. (2003); McAleer and Medeiros (2008); Amado and Teräsvirta (2013); Silvennoinen and Teräsvirta (2013). We underline that the role of the dynamic asymmetry and, in particular, its intersection with stochastic/time-varying volatility has never been investigated.

Fourth, dissectangling the role of parameter uncertainty is not easy. A very simplistic scheme can be set as follows: pro-cyclical variables seem not be influenced more than a small part of the measures adopted. On the opposite side, uncertainty downgrades the majority of dynamic asymmetric models of anti-cyclical proxies, so that the STARs return often preferable.

Finally, our forecast comparisons are based on statistical tests originally developed for linear models. Very little is known about the inference on uncertain environment if dynamic asymmetry is assumed. The fluctuation test by Giacomini and Rossi (2010) was supposed clarify our findings. Unfortunately, according to our preliminary results (not shown), the fluctuation tests seems to over-reject the null hypothesis, also when linear autoregression is a reasonable hypothesis; thus, we prefer to postpone this important issue for further methodological investigation.

5 Conclusions

The Generalized Logistic function is applied to STAR family of models as simple, statistically feasible way to capture the dynamic asymmetry in the conditional mean

[12]We do not show the results due to space limits. Further results can be provided under request.

of a time series. The resulting GSTAR model, due to its logarithmic (exponential) rescaling imposed by h-functions (3) - (4), ensures the smoothness of the transition function by construction without requiring further efforts for what concerns identification and estimation.

Our applications to US macroeconomic series enable us to see the business cycle in a new light, specially in view of the most recent literature. The US business cycle is clearly characterized by dynamically asymmetric cycles, and a properly specified GSTAR model leads to substantial gain in point forecasting ability.

The dynamic asymmetry affects almost exclusively the conditional mean of the process. Its detection in higher conditional moments and its statistical treatment in unstable forecasting environments are still unexplored. We think a proper treatment of these features would possibly reduce the major idiosyncrasies previously described. We encourage further research effort in this direction, since the high flexibility of the GSTAR model seems a valid tool to model and forecast other prominent, nonlinearly behaving-economic variables like monetary aggregates and financial time series.

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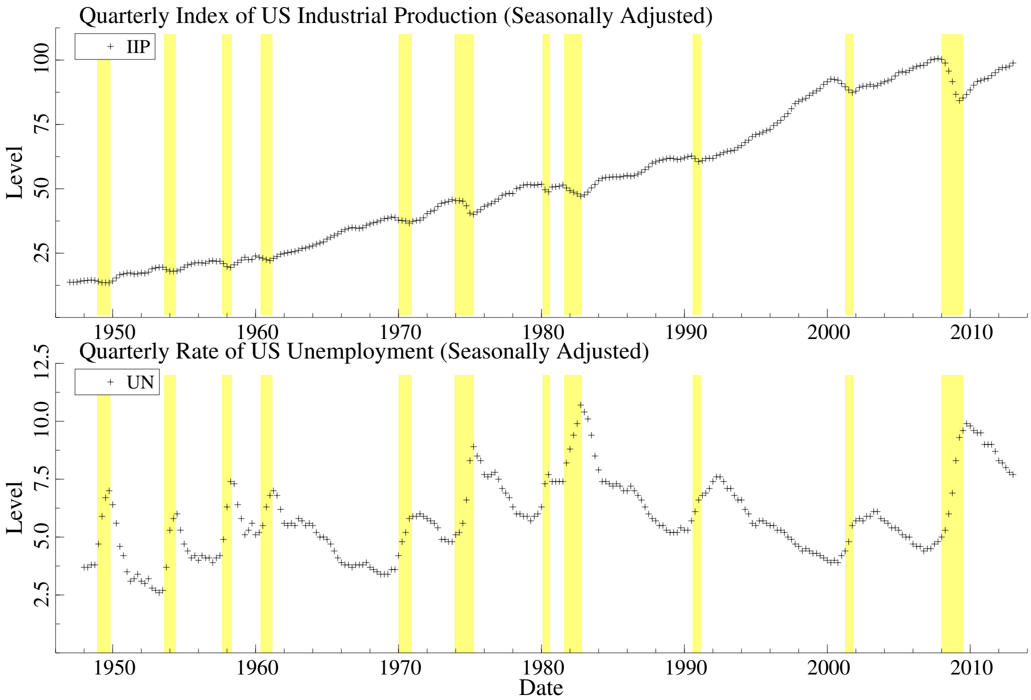
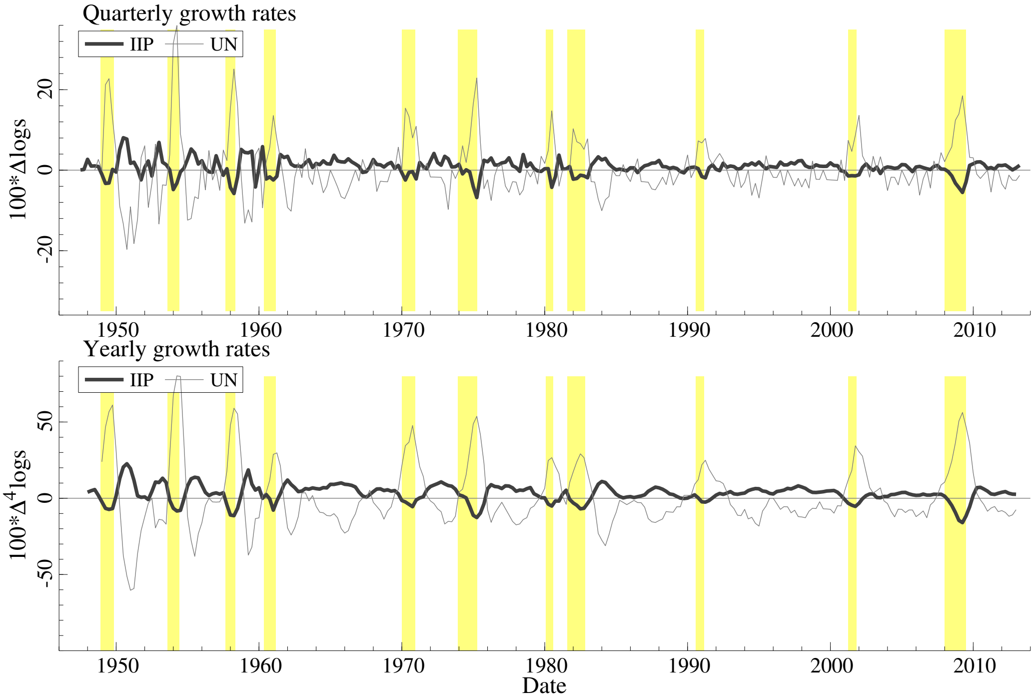
Tables and Graphs

Figure 1: The US quarterly data

(a) Data in levels



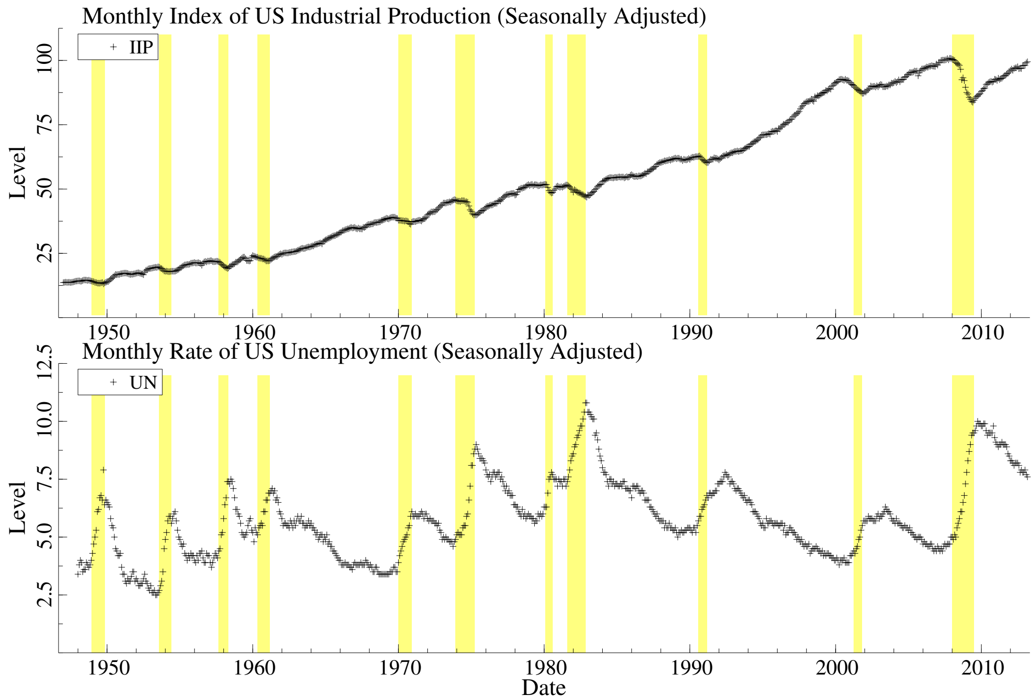
(b) Data in growth rates



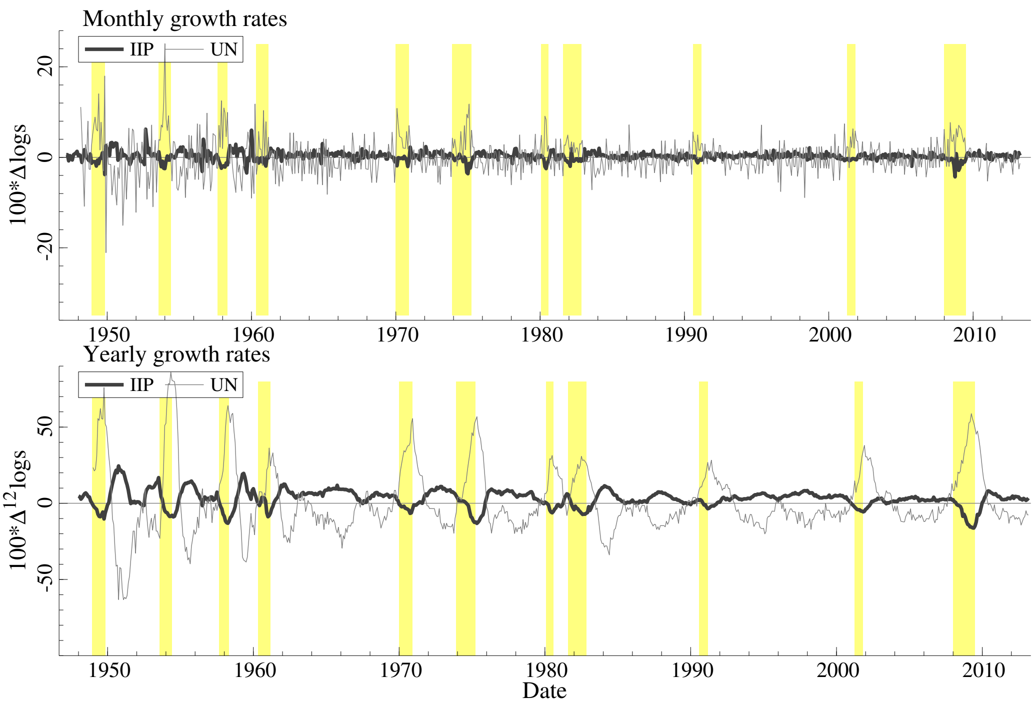
NOTE: This figure plots the quarterly data on US index of industrial production (IIIP) and unemployment (UN) used to illustrate the performances of GSTAR model in Section 4. In particular, panel (a) plots the series in levels, while panel (b) plots the same series in quarterly (upper sub-panel) and yearly growth rates (lower sub-panel), respectively. The bands in yellow are the NBER recession dates.

Figure 2: The US monthly data

(a) Data in levels



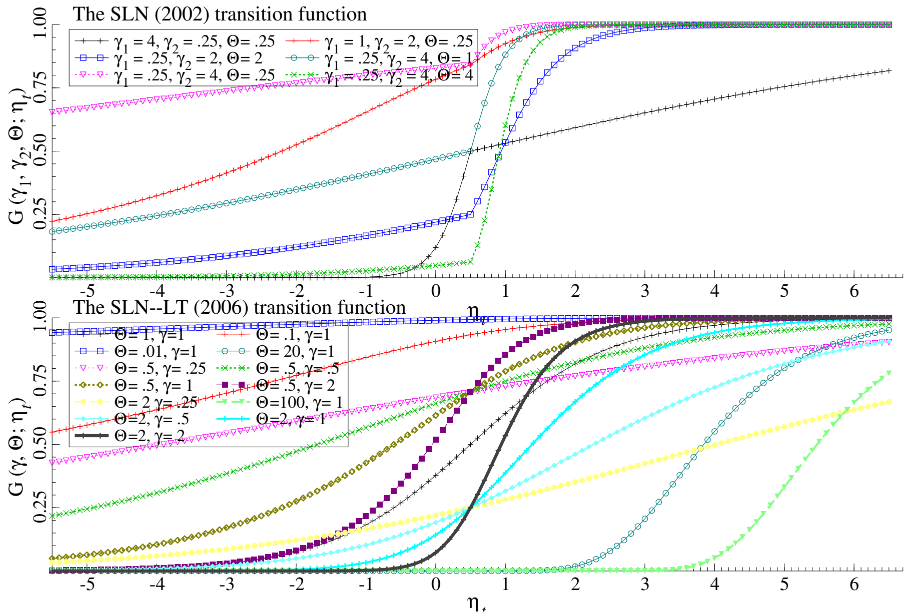
(b) Data in growth rates



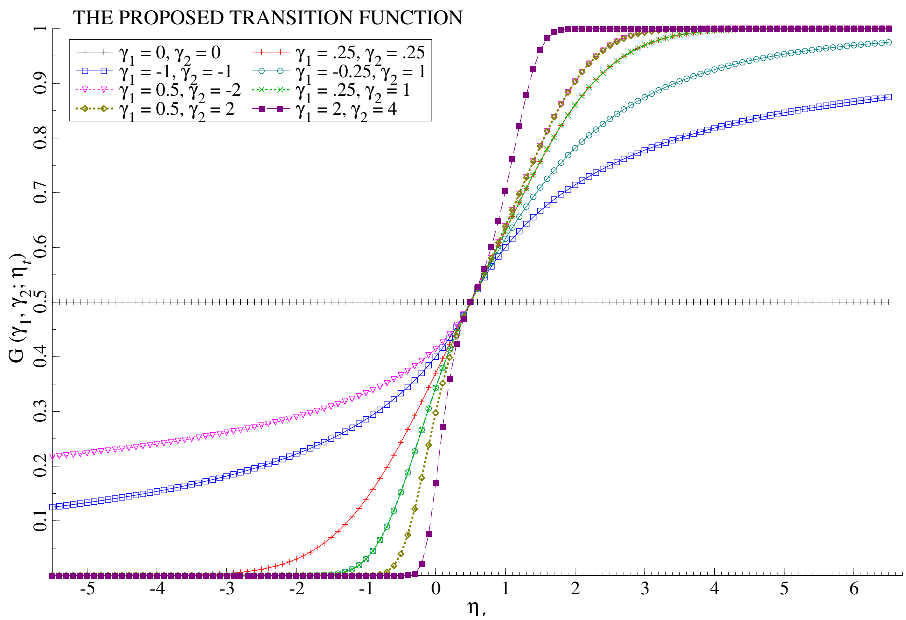
NOTE: This figure plots the monthly data on US index of industrial production (IIIP) and unemployment (UN) used to illustrate the performances of GSTAR model in Section 4. In particular, panel (a) plots the series in levels, while panel (b) plots the same series in monthly (upper sub-panel) and yearly growth rates (lower sub-panel), respectively. The bands in yellow are the NBER recession dates.

Figure 3: Types of asymmetry in the transition functions.

(a) Asymmetric transition functions



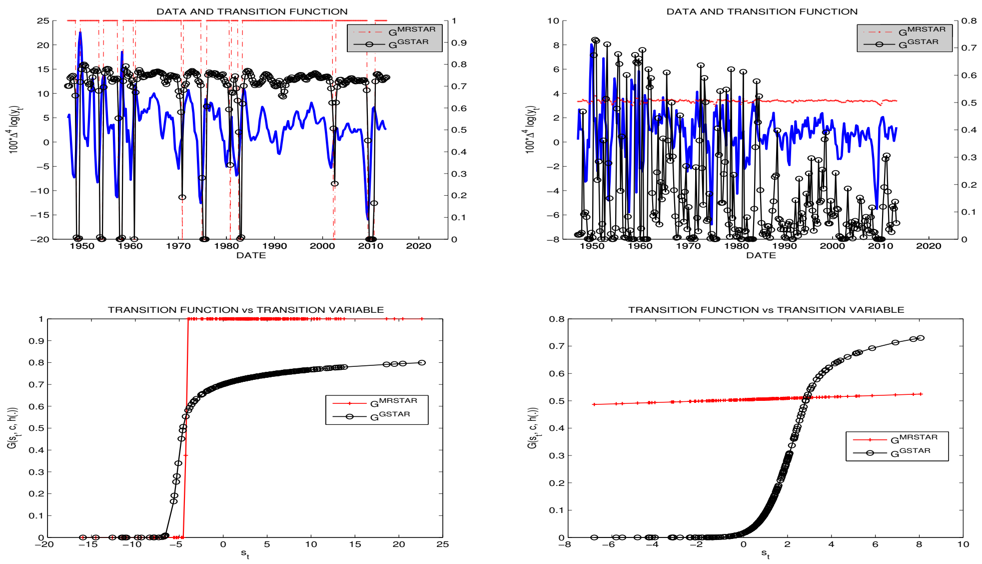
(b) Dynamically asymmetric transition functions



NOTE: This figure illustrates several different parametrizations of asymmetric transition function for a STAR model. In particular, panel (a) plots the asymmetric ones currently available in literature, namely the Sollis et al. (2002) and the Sollis et al. (1999)–Lundbergh and Teräsvirta (2006) models; panel (b) plots the dynamically asymmetric transition function here proposed.

Figure 4: Estimated transition functions specifications for quarterly IIP.

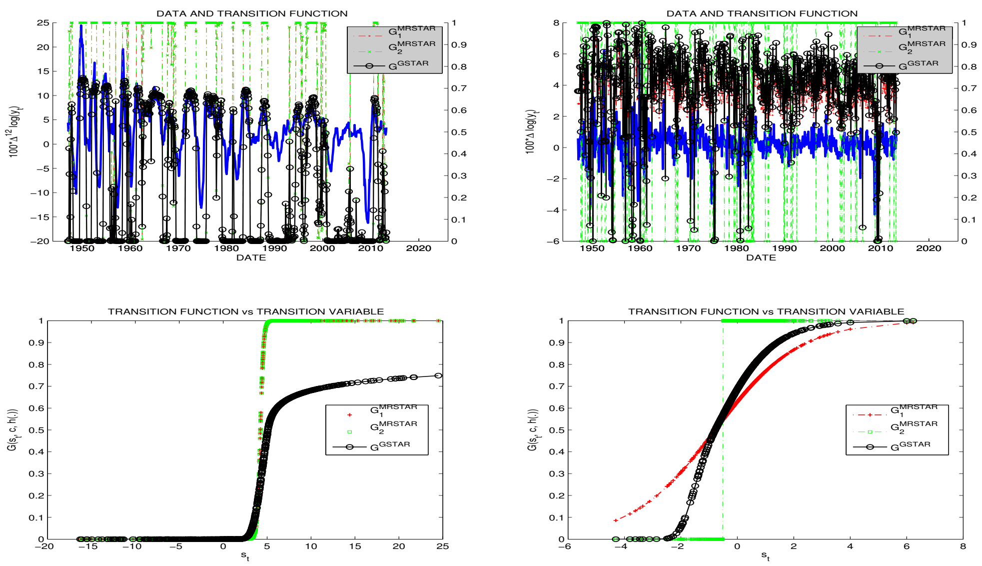
(a) Yealy growth rate **(b)** Monthly growth rate



NOTE: This figures plots the estimated transition functions of MRSTAR and GSTAR models estimated from quarterly IIP series. The left-had side shows the results for yearly growth rates, while right-had side the monthly growth rates; upper panels plots data with transition functions versus time, while in bottom panels the same transitions are shown versus the transition variable.

Figure 5: Estimated transition functions specifications monthly IIP.

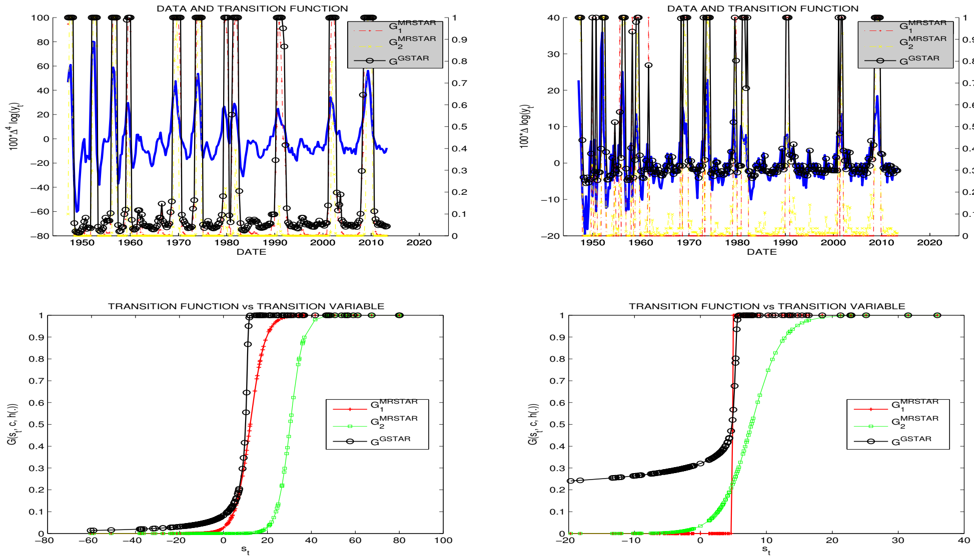
(a) Yealy growth rate **(b)** Monthly growth rate



NOTE: This figures plots the estimated transition functions of MRSTAR and GSTAR models estimated from monthly IIP series. The left-had side shows the results for yearly growth rates, while right-had side the monthly growth rates; upper panels plots data with transition functions versus time, while in bottom panels the same transitions are shown versus the transition variable.

Figure 6: Estimated transition functions for quarterly UN.

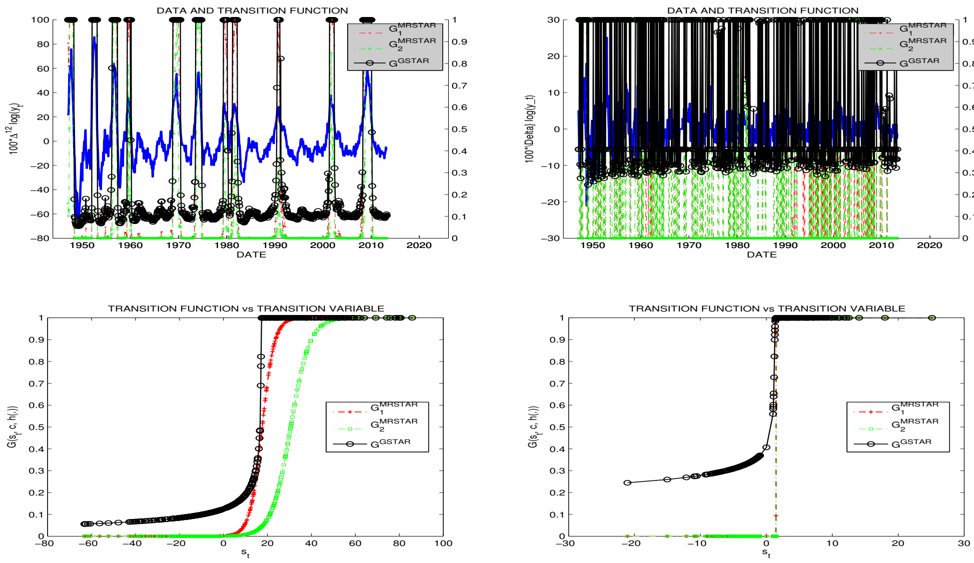
(a) Yealy growth rate **(b)** Monthly growth rate



NOTE: This figures plots the estimated transition functions of MRSTAR and GSTAR models estimated from quarterly UN series. The left-had side shows the results for yearly growth rates, while right-had side the monthly growth rates; upper panels plots data with transition functions versus time, while in bottom panels the same transitions are shown versus the transition variable.

Figure 7: Estimated transition functions for monthly UN.

(a) Yealy growth rate **(b)** Monthly growth rate



NOTE: This figures plots the estimated transition functions of MRSTAR and GSTAR models estimated from monthly UN series. The left-had side shows the results for yearly growth rates, while right-had side the monthly growth rates; upper panels plots data with transition functions versus time, while in bottom panels the same transitions are shown versus the transition variable.

Table 1: Parameter estimation and diagnostics of the (G)STAR model for US data.

Linearity and Asymmetry Tests																
s	y_{t-3}		y_{t-3}		y_{t-4}		y_{t-4}		y_{t-1}		y_{t-1}		y_{t-3}			
Linearity	0.0037		0.0662		0.0025		0.0708		0.0157		0.0225		0.0343		0.0370	
Dynamic Symmetry (1)	0.0005		0.0447		0.0159		0.0787		0.0103		0.0035		0.0009		0.0416	
Estimation																
	IIP								UN							
Parameter	$q\Delta^4 \log$ IIP		$q\Delta \log$ IIP		$m\Delta^{12} \log$ IIP		$m\Delta \log$ IIP		$q\Delta^4 \log$ UN		$q\Delta \log$ UN		$m\Delta^{12} \log$ UN		$m\Delta \log$ UN	
	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE
ϕ_0	2.6488	0.4734	0.4536	0.0378	0.1806	0.0167	0.1087	0.1364	-1.7534	0.4093	0.2456	0.1984	-1.2428	0.2972	-0.0004	0.1100
ϕ_1	1.7250	0.0557	0.7788	0.0186	1.2750	0.0114	0.2715	0.0266	1.0400	0.0315	0.3765	0.0521	0.7849	0.0172	-0.0790	0.0488
ϕ_2	-1.0922	0.0947	-0.4036	0.0227	-0.2494	0.0185	-0.0690	0.0349	-0.1734	0.0376	0.3968	0.0812	0.2465	0.0161	0.0621	0.0538
ϕ_3	0.0300	0.0945	0.3804	0.0274	0.0104	0.0183	0.3000	0.0505	-0.1543	0.0331	-0.2444	0.0602	-0.0452	0.0162	-0.0034	0.0543
ϕ_4	0.3109	0.0470	-0.4777	0.0197	0.0245	0.0184	-0.0330	0.0333	-0.0040	0.0183	-0.2386	0.0245	-0.0445	0.0163	0.0091	0.0316
ϕ_5	.	.	.	.	-0.1131	0.0189	-0.1199	0.0233	.	.	.	.	0.0501	0.0165	0.3258	0.0875
ϕ_6	.	.	.	.	0.0262	0.0190	-0.1180	0.0564	.	.	.	.	0.0005	0.0175	0.2151	0.0815
ϕ_7	.	.	.	.	-0.0781	0.0117	-0.0030	0.0311	.	.	.	.	-0.1180	0.0116	0.0792	0.0526
θ_{10}	-3.2434	0.7710	1.6501	0.8383	0.7657	0.2203	0.0432	0.1960	12.8150	0.6691	-1.0121	0.5547	6.4117	0.4173	-0.0763	0.1807
θ_{11}	-0.5826	0.0945	-0.8773	0.3813	-0.2062	0.0540	0.0754	0.0380	12.8150	0.0423	0.6308	0.0607	0.1350	0.0256	0.1780	0.0524
θ_{12}	0.6680	0.1440	0.4802	0.1125	0.3835	0.0954	0.2515	0.0434	0.2502	0.0568	-1.0211	0.0867	-0.0777	0.0298	0.1897	0.0575
θ_{13}	0.3037	0.1386	-0.6568	0.2006	-0.1034	0.0210	-0.3028	0.0598	-0.4291	0.0544	0.6534	0.0711	-0.0034	0.0300	0.2239	0.0597
θ_{14}	-0.6241	0.0907	0.5853	0.1554	-0.3570	0.0346	0.1029	0.0262	-0.0629	0.0353	0.1797	0.0511	-0.0405	0.0298	0.0982	0.0372
θ_{15}	.	.	.	.	-0.0068	0.0454	0.0821	0.0349	.	.	.	.	0.0793	0.0293	-0.3322	0.0892
θ_{16}	.	.	.	.	0.2307	0.0405	0.1641	0.0430	.	.	.	.	-0.1915	0.0306	-0.3174	0.0833
θ_{17}	.	.	.	.	0.0459	0.0109	0.0711	0.0196	.	.	.	.	-0.0186	0.0230	-0.1866	0.0566
γ_1	-3.2500	0.3016	-2.7500	0.2695	-4.0000	0.1633	0.0100	0.1897	1.0000	0.1691	5.2500	0.3547	5.2500	0.4173	7.0000	0.4229
γ_2	0.7500	0.1006	0.2500	0.2768	0.5000	0.1874	1.2366	0.1404	-1.0000	0.0423	-2.0000	0.0607	-1.7500	0.0256	-3.9739	0.4256
c_1	-4.5728	0.1079	2.8248	0.3198	5.0854	0.1284	-0.7846	0.0001	10.0658	0.0568	4.9501	0.0867	16.6762	0.0298	0.8749	0.0995
Dyagnostics																
No Error Autocorrelation	0.0590		0.0491		0.0731		0.0459		0.0395		0.0521		0.9504		0.9449	
No Add. Asymmetry	0.8530		0.2490		1.0000		1.0000		0.2105		0.1690		1.0000		0.9995	
Param. Constancy	0.9314		0.9900		0.9010		0.9592		0.8425		0.8257		0.9884		1.0000	

NOTE: This table summarizes the main findings in applications of our GSTAR model to the US data in growth rates. Namely, in the first panel reports the values of each parameter with standard error. The second panel reports the $p-$values of the Luukkonen et al. (1988) test for the null hypothesis of linearity as well as the proposed test corrsponding to equation 13 corresponding to the transition variable s_{t-d} where delay d is selected to minimize the $p-$value. Finally, the third panel reports the p-values of the three diagnostic tests explained in the Supplement.

Table 2: GSTAR versus linear and nonlinear symmetric models: comparison of predictive performances in quarterly US data.

Point predictive performances							
Forecast horizon	Forecast Error Measure	AR		(MR)STAR		GSTAR	
		IIP	UN	IIP	UN	IIP	UN
1	MFE	-0.1823	-0.8710	-0.1417	**-0.0456**	**-0.1070**	-0.8698
2		-0.1928	-0.9323	-0.1437	**-0.0525**	**-0.1094**	-0.9255
4		-01969	-0.9528	-0.1377	**-0.0831**	**-0.1076**	-1.0273
8		-0.1991	-1.1300	-0.1616	**-0.1271**	**-0.1349**	-1.1687
1	sMAE	0.0077	0.0669	**0.0030**	0.0676	0.0032	**0.0630**
2		0.0079	0.0692	**0.0030**	0.0745	0.0032	**0.0665**
4		0.0085	0.0708	**0.0031**	0.0792	0.0033	**0.0626**
8		0.0087	0.0732	**0.0032**	0.0737	0.0034	**0.0650**
1	mRAE	1.0000	1.0000	1.3433	0.4214	**1.2074**	**0.3063**
2		1.0000	1.0000	1.4061	0.3997	**1.3400**	**0.3099**
4		1.0000	1.0000	1.6696	0.4139	**1.6212**	**0.3559**
8		1.0000	1.0000	2.6900	0.4817	**2.5691**	**0.3879**
1	RMSFE	0.1154	1.7223	0.1151	1.5889	**0.1136**	**1.0372**
2		0.1159	1.8310	0.1158	1.5991	**0.1144**	**1.0427**
4		0.1172	1.8969	0.1171	1.6197	**0.1158**	**1.0542**
8		0.1224	1.9344	**0.1197**	1.6626	0.1285	**1.0805**
Density predictive performances							
1	LogS	**0.0104**	0.0179	0.0105	0.0176	0.0110	**0.0166**
2		0.0105	0.0182	**0.0105**	0.0176	0.0109	**0.0166**
4		0.0107	0.0182	**0.0106**	0.0175	0.0112	**0.0172**
8		**0.0107**	0.0184	0.0110	0.0175	0.0123	**0.0172**
1	QS	0.2138	0.0651	0.2202	0.0647	**0.2131**	**0.0637**
2		0.2201	0.0654	0.2191	0.0650	**0.2111**	**0.0639**
4		0.2252	0.0659	0.2210	0.0650	**0.2113**	**0.0639**
8		0.2252	0.0662	0.2219	0.0653	**0.2152**	**0.0642**
1	CRPS	7.0738	31.5422	**7.0256**	31.5500	7.0655	**31.5100**
2		7.1141	31.7299	**7.0750**	31.7077	7.1123	**31.7377**
4		7.2140	32.2400	**7.1740**	32.0356	7.2132	**32.0383**
8		7.4200	32.7444	**7.3708**	32.7216	7.4222	**32.7219**
1	qS	0.2092	0.0421	0.2088	0.0382	**0.2038**	**0.0376**
2		0.2103	0.0489	0.2085	0.0449	**0.2036**	**0.0443**
4		0.2118	0.0528	0.2066	**0.0524**	**0.2020**	0.0584
8		0.2118	**0.0772**	0.2048	0.0803	**0.2004**	0.0803

NOTE: This table reports the point (first panel) and density (second panel) predictive performances of linear (AR) nonlinear symmetric (MRSTAR) and dynamic asymmetric (GSTAR) models according to different measures and forecasting horizons with testing period from 1982:Q3 to 2013Q1 for IIP and from 1983:Q3 to 2013Q1 for UN. The smallest values for each forecasting horizon are in bold.

Table 3: GSTAR versus linear and nonlinear symmetric models: comparison of predictive performances in monthly US data

		Point predictive performances					
Forecast horizon	Forecast Error Measure	AR		(MR)STAR		GSTAR	
		IIP	UN	IIP	UN	IIP	UN
1		**-0.0011**	**0.0009**	-0.0065	0.0156	-0.0093	0.0095
3	MFE	-0.1108	0.0340	**-0.0071**	0.0168	-0.0098	**0.0108**
6		-0.3024	0.0607	**-0.0072**	0.0150	-0.0099	**0.0093**
12		-1.4908	0.1117	**-0.0068**	0.0163	-0.0094	**0.0107**
1		0.0011	**0.0058**	0.0011	0.0061	**0.0011**	0.0060
3	sMAE	0.0037	0.0087	0.0011	0.0059	**0.0011**	**0.0058**
6		0.0070	0.0128	0.0012	0.0056	**0.0012**	**0.0055**
12		0.0124	0.0190	0.0012	0.0050	**0.0012**	**0.0049**
1		1.0000	1.0000	1.0324	1.0224	**1.0290**	**1.0107**
3	mRAE	1.0000	1.0000	1.1560	1.0552	**1.1241**	**1.0369**
6		1.0000	1.0000	1.4886	1.2189	**1.4546**	**1.1693**
12		1.0000	1.0000	**2.0115**	**1.4965**	2.1142	1.5085
1		0.0698	0.2856	0.0051	0.0178	**0.0042**	**0.0176**
3	RMSFE	0.2894	0.4234	0.0052	0.0179	**0.0043**	**0.0177**
6		0.7552	0.5915	0.0052	0.0181	**0.0046**	**0.0179**
12		0.7638	0.8545	0.0055	0.0184	**0.0049**	**0.0182**
		Density predictive performances					
1		0.0020	**0.0031**	0.0017	0.0033	**0.0014**	0.0036
3	LogS	0.0018	**0.0034**	0.0017	0.0036	**0.0014**	0.0035
6		0.0021	0.0035	0.0017	0.0036	**0.0016**	**0.0035**
12		0.0032	0.0036	**0.0017**	0.0036	0.0021	**0.0035**
1		2.1568	**0.7986**	**2.1303**	0.8511	2.1689	0.8683
3	QS	2.1644	**0.7998**	**2.1386**	0.8537	2.1821	0.8698
6		2.1665	**0.8125**	**2.1391**	0.8550	2.1895	0.8698
12		2.2024	0.6741	**2.1479**	0.6412	2.1921	**0.6354**
1		**0.2142**	1.6750	0.2170	**1.6090**	0.2201	1.6094
3	CRPS	**0.2161**	1.6850	0.2189	**1.6193**	0.2224	1.6197
6		**0.2188**	1.6983	0.2212	**1.6317**	0.2252	1.6344
12		**0.2255**	1.7295	0.2279	**1.6594**	0.2318	1.6617
1		0.0094	-0.0713	0.0089	**-0.0727**	**0.0087**	-0.0723
3	qS	0.0093	-0.0706	**0.0085**	**-0.0721**	0.0086	-0.0717
6		0.0092	-0.0695	**0.0083**	**-0.0708**	0.0085	-0.0705
12		0.0087	-0.0667	**0.0071**	**-0.0682**	0.0080	-0.0679

NOTE: This table reports the point (first panel) and density (second panel) predictive performances of linear (AR) nonlinear symmetric (MRSTAR) and dynamic asymmetric (GSTAR) models according to different measures and forecasting horizons with testing period from 1982:M9 to 2013:M3 for IIP and from 1983:M9 to 2013:M3 for UN. The smallest values for each forecasting horizon are in bold.

Table 4: GSTAR versus linear and nonlinear symmetric models under uncertainty: comparison of predictive performances in quarterly US data

Forecast horizon	Forecast Error Measure	AR		(MR)STAR		GSTAR	
		IIP	UN	IIP	UN	IIP	UN
Point predictive performances							
1	MFE	**-2.4600**	-1.2362	-2.4670	**-1.1701**	-2.4685	-1.3015
2		**-2.4659**	-1.2930	-2.4632	**-1.2361**	-2.4686	-1.3664
4		**-2.4682**	-1.4295	-2.4608	**-1.3745**	-2.4427	-1.5040
8		-2.4682	-1.6072	-2.4586	**-1.5868**	**-2.4391**	-1.7101
1	sMAE	0.0204	**0.1101**	0.0201	0.1266	**0.0200**	0.1120
2		0.0204	**0.1062**	0.0202	0.1208	**0.0202**	0.1074
4		0.0210	**0.0975**	0.0205	0.1103	**0.0205**	0.0992
8		0.0217	**0.0902**	0.0215	0.0992	**0.0214**	0.0907
1	mRAE	1.0000	1.0000	**1.0000**	1.0035	1.0001	**1.0017**
2		1.0000	1.0000	1.0042	**1.0011**	**1.0038**	1.0025
4		1.0000	1.0000	**1.0069**	**1.0016**	1.0071	1.0025
8		1.0000	1.0000	**1.0037**	**1.0021**	1.0039	1.0034
1	RMSFE	**0.3707**	**1.1720**	0.3756	1.2322	0.3751	1.2337
2		**0.3723**	**1.1786**	0.3775	1.2387	0.3771	1.2403
4		**0.3761**	**1.1908**	0.3805	1.2518	0.3800	1.2535
8		**0.3835**	**1.2201**	0.3888	1.2820	0.3882	1.2841
Density predictive performances							
1	LogS	0.0136	**0.0169**	0.0135	0.0176	**0.0130**	0.0181
2		0.0135	**0.0169**	0.0134	0.0176	**0.0130**	0.0181
4		0.0135	**0.0168**	0.0133	0.0175	**0.0129**	0.0180
8		0.0133	**0.0167**	0.0131	0.0175	**0.0128**	0.0180
1	QS	0.2244	0.0635	0.2242	**0.0634**	**0.2240**	0.0639
2		0.2243	0.0638	0.2240	**0.0637**	**0.2238**	0.0639
4		0.2243	0.0640	0.2244	**0.0637**	**0.2239**	0.0647
8		0.2245	0.0652	0.2245	0.0652	**0.2244**	**0.0647**
1	CRPS	6.2854	**30.1794**	6.0036	31.5500	**5.9310**	30.4696
2		6.3291	**30.3430**	6.0443	31.7077	**5.9689**	30.6087
4		6.4230	**30.6666**	6.1338	32.0356	**6.0561**	30.9277
8		6.6149	**31.3386**	6.3109	32.7216	**6.2307**	31.6032
1	qS	**0.0440**	0.0093	0.0463	**0.0075**	0.2330	0.0509
2		**0.0440**	0.0105	0.0463	**0.0089**	0.2327	0.0576
4		**0.0437**	0.0133	0.0460	**0.0117**	0.2311	0.0716
8		**0.0432**	0.0170	0.0456	**0.0161**	0.2291	0.0928

NOTE: This table reports the point (first panel) and density (second panel) predictive performances of linear (AR) nonlinear symmetric (MRSTAR) and dynamic asymmetric (GSTAR) models accounting for uncertainty via block-boostrap algorithm according to different measures and forecasting horizons with testing period from 1982:Q3 to 2013Q1 for IIP and from 1983:Q3 to 2013Q1 for UN. The smallest values for each forecasting horizon are in bold.

Table 5: GSTAR versus linear and nonlinear symmetric models under uncertainty: comparison of predictive performances in monthly US data.

Forecast horizon	Forecast Error Measure	AR		(MR)STAR		GSTAR	
		IIP	UN	I5P	UN	IIP	UN
Point predictive performances							
1		0.7134	-0.0309	0.5894	-0.0308	**0.5781**	**0.0008**
3	MFE	0.8400	**-0.0006**	0.7628	-0.0205	**0.5657**	0.0465
6		**0.5048**	**-0.0020**	0.7489	-0.0166	0.6159	-0.0074
12		0.8711	-0.0530	0.8340	-0.0129	**0.7099**	**-0.0090**
1		0.0057	0.0308	**0.0020**	**0.0144**	0.0058	0.0272
3	sMAE	0.0056	0.0271	**0.0020**	**0.0137**	0.0056	0.0318
6		0.0056	0.0253	**0.0021**	**0.0131**	0.0061	0.0258
12		0.0059	0.0261	**0.0022**	**0.0124**	0.0056	0.0231
1		1.0000	1.0000	0.6814	**0.0063**	**0.6143**	0.0173
3	mRAE	1.0000	1.0000	1.1191	0.1223	**0.9089**	**0.1175**
6		1.0000	1.0000	3.0881	0.1245	**0.9029**	**0.1701**
12		1.0000	1.0000	5.1183	0.1421	**0.9937**	**0.2119**
1		0.0641	0.0780	**0.0368**	**0.0603**	0.0598	0.0774
3	RMSFE	0.0662	0.0789	**0.0329**	**0.0604**	0.0543	0.0781
6		0.0489	0.0784	**0.0425**	**0.0606**	0.0510	0.0786
12		0.0706	0.0797	**0.0483**	**0.0612**	0.0625	0.0800
Density predictive performances							
1		0.0019	0.0036	**-0.0001**	**0.0034**	0.0020	0.0037
3	LogS	0.0018	0.0035	**-0.0001**	**0.0034**	0.0023	0.0039
6		0.0018	0.0035	**-0.0001**	**-0.0166**	0.0029	0.0040
12		0.0018	0.0037	**-0.0001**	**0.0034**	0.0038	0.0044
1		2.2142	0.8870	2.5261	1.0937	**2.2040**	**0.8763**
3	QS	2.2146	0.8879	2.4917	1.0995	**2.2086**	**0.8769**
6		2.2234	0.8876	**2.1880**	1.0929	2.2100	**0.8773**
12		2.2250	0.8920	2.2437	1.0095	**2.2195**	**0.8819**
1		**-0.4926**	1.6349	0.1968	**0.8900**	0.2219	1.6349
3	CRPS	0.2241	1.6431	**0.1974**	**0.8922**	0.2241	1.6431
6		0.2273	1.6579	**0.1982**	**0.8954**	0.2273	1.6579
12		0.2337	1.6865	**0.1999**	**0.9018**	0.2337	1.6865
1		0.0017	**-0.0157**	**-0.0030**	-0.0142	0.0019	-0.0145
3	qS	0.0017	**-0.0155**	**-0.0030**	-0.0141	0.0019	-0.0144
6		0.0019	**-0.0153**	**-0.0030**	-0.0140	0.0020	-0.0141
12		0.0021	**-0.0146**	**-0.0031**	-0.0138	0.0025	-0.0136

NOTE: This table reports the point (first panel) and density (second panel) predictive performances of linear (AR) nonlinear symmetric (MRSTAR) and dynamic asymmetric (GSTAR) models accounting for uncertainty via block-boostrap algorithm according to different measures and forecasting horizons with testing period from 1982:M9 to 2013:M3 for IIP and from 1983:M9 to 2013:M3 for UN. The smallest values for each forecasting horizon are in bold.

Table 6: Comparison of predictive ability tests of GSTAR versus linear and symmetric models in quarterly US data.

	MR-STAR vs AR				GSTAR vs MR-STAR			
SERIES	IIP		UN		IIP		UN	
h	Diebold-Mariano							
1	0.1196		0.1668		0.1165		0.2260	
2	0.0890		0.2550		0.1170		0.2259	
4	0.1011		0.2540		0.1978		0.2841	
8	0.2104		0.3054		0.2080		0.3270	
h	Giacomini-Whight							
1	0.0610		0.0568		0.0327		0.0167	
2	0.0890		0.0761		0.1366		0.0411	
4	0.1244		0.0034		0.2028		0.2914	
8	0.3351		0.2804		0.3965		0.5199	
Scoring Rule	Amisano-Giacomini							
h	IP		UN		IP		UN	
QSR	*t*-statistic	*p*-val	*t*-statistic	*p*-val	*t*-statistic	*p*-val	*t*-statistic	*p*-val
1	$1.9e^{e-5}$	0.4600	0.0015	0.4994	$2.9e^{4}$	0.5567	$6.0e^{-4}$	0.4999
2	$1.9e^{-5}$	0.4975	0.0015	0.4994	$2.9e^{4}$	0.5553	$6.0e^{-4}$	0.4999
4	$2.9e{-}7$	0.5120	0.0016	0.4993	$2.9e^{4}$	0.5312	$4.1e^{-5}$	0.5998
8	$2.9e^{-9}$	0.5169	0.0016	0.4993	$2.3e^{4}$	0.5622	$9.2e^{-5}$	1.0000
LogS								
1	-16.2383	1.0000	-2017.1657	1.0000	-0.3535	0.6380	-3.9e^{3}	1.0000
2	-16.3005	1.0000	-2025.0146	1.0000	-0.3548	0.6385	-2.2e^{4}	1.0000
4	-16.4249	1.0000	-2040.7124	1.0000	-0.3576	0.6395	-5.1e^{5}	1.0000
8	-16.6738	1.0000	-2072.1083	1.0000	-0.3630	0.6415	-6.2e^{6}	1.0000
CRPS								
1	2.2 e^{4}	$<$0.001	1.4 e^{07}	$<$0.001	2.4 e^{4}	$<$0.001	2.4 e^{5}	$<$0.001
2	2.1 e^{4}	$<$0.001	2.2e^{06}	$<$0.001	2.0 e^{5}	$<$0.001	2.4 e^{4}	$<$0.001
4	2.4 e^{4}	$<$0.001	2.2 e^{06}	$<$0.001	2.0 e^{5}	$<$0.001	2.4 e^{4}	$<$0.001
8	2.5 e^{4}	$<$0.001	2.4 e^{06}	$<$0.001	2.0 e^{6}	$<$0.001	2.4 e^{4}	$<$0.001
QuantS								
1	3.5e^{5}	$<$0.001	3.2e^{4}	$<$0.001	-7e^{5}	1.0000	-3.5e^{6}	1.0000
2	5.2e^{6}	$<$0.001	4.9e^{4}	$<$0.001	-2e^{5}	1.0000	-8.3e^{6}	1.0000
4	5.7e^{6}	$<$0.001	4.9e^{4}	$<$0.001	-1.4e^{5}	1.0000	-9.0e^{7}	1.0000
8	3.8e^{6}	$<$0.001	4.9e^{4}	$<$0.001	-2e^{7}	1.0000	-6.4e^{7}	1.0000

NOTE: This table reports the results of equal predictive ability tests for GSTAR versus AR and MRSTAR models in quarterly US data in growth rates for the testing period from 1982:Q3 to 2013:Q1. All the tests consider density forecasts generated from real data, according to the model estimated in Table **??**. In the Amisano-Giacomini test, the GSTAR has the role of $\bar{S}^f$ and the benchmark (MR-ST)AR density forecast the role of $\bar{S}^g$. Since here LogS has positive orientation, if t-statistic is positive, f is preferred; the weight is assumed 1.

Table 7: Comparison of predictive ability tests of GSTAR versus linear and symmetric models in monthly US data.

	MR-STAR vs AR				GSTAR vs MR-STAR			
SERIES	IIP		UN		IIP		UN	
h	Diebold-Mariano							
1	0.0000		0.0000		0.0000		0.0000	
2	0.0003		0.0005		0.0000		0.0000	
4	0.0012		0.0019		0.0000		0.0000	
12	0.0026		0.0038		0.0000		0.0002	
h	Giacomini-Whight							
1	0.0000		0.0000		0.0000		0.0000	
2	0.0000		0.0018		0.0000		0.0000	
4	0.0066		0.0072		0.0040		0.0215	
12	0.0106		0.0133		0.0170		0.0420	
Scoring Rule	Amisano-Giacomini							
h	IIP		UN		IIP		UN	
QSR	t-statistic	p-val	t-statistic	p-val	t-statistic	p-val	t-statistic	p-val
1	5.1057	<0.001	4.8320	<0.001	-5.1057	1.0000	4.2904	<0.001
2	0.3733	0.7067	4.9038	<0.001	0.3733	0.3545	4.5631	<0.001
4	-0.5440	0.2150	5.1023	0.0083	0.5440	0.2932	4.6200	0.0250
12	5.0349	<0.001	5.7302	0.0152	-5.0349	1.0000	-1.0039	0.9231
LogS								
1	650.28	1.0000	602.34	1.0000	649.281	<0.001	43.764	0.6773
2	-192.36	0.8899	-132.48	0.7530	-192.364	1.0000	30.450	0.3502
4	$-3.7e^3$	1.0000	$-2.2e03$	1.0000	$3.7e03$	<0.001	12.304	0.1339
12	-641.28	1.0000	-459.23	1.0000	641.2803	<0.001	12.439	0.1010
CRPS								
1	-0.41249	0.6599	-0.2995	0.3028	0.41249	0.3400	-0.3702	0.5265
2	-1.2273	0.1100	-0.4102	0.2504	-1.2273	0.8899	-0.7894	0.2028
4	2.2718	0.0116	-0.1501	0.2050	-2.2718	0.9883	-1.1490	0.0922
12	-0.4067	0.6578	-0.9501	0.5699	0.4067	0.3421	-1.4670	0.0579
QuantS								
1	7.3000	1.0000	5.7031	0.9940	-7.3029	1.0000	4.2499	0.3401
2	7.0429	1.0000	5.2301	0.9024	7.0429	1.0000	3.2993	0.2370
4	8.7815	<0.001	2.1027	0.4501	-8.7815	1.0000	2.3409	0.1502
12	7.1988	<0.001	1.8235	0.3501	-7.1988	1.0000	2.9302	0.0829

NOTE: This table reports the results of equal predictive ability tests for GSTAR versus AR and MRSTAR models in monthly US data in growth rates for the testing period from 1982:Q3 to 2013:Q1. All the tests consider density forecasts generated from real data, according to the model estimated in Table **??**. In the Amisano-Giacomini test, the GSTAR has the role of $\bar{S}^f$ and the benchmark (MR-ST)AR density forecast the role of $\bar{S}^g$. Since here LogS has positive orientation, if t-statistic is positive, f is preferred; the weight is assumed 1.

A Appendix

A.1 Estimation

Following Leybourne et al. (1998), estimation is done by concentrating the Sum of Square Residuals function with respect to $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$, that is minimizing:

$$SSR = \sum_{t=1}^{T} \left(y_t - \hat{\boldsymbol{\psi}}' \mathbf{x}_\mathbf{t}' \right)^2, \tag{16}$$

where:

$$\hat{\boldsymbol{\psi}} = [\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\theta}}] = \left(\sum_{t=1}^{T} \mathbf{x}_\mathbf{t}'(\boldsymbol{\gamma}, \mathbf{c}) \mathbf{x}_\mathbf{t}(\boldsymbol{\gamma}, \mathbf{c}) \right)^{-1} \left(\sum_{t=1}^{T} \mathbf{x}_\mathbf{t}'(\boldsymbol{\gamma}, \mathbf{c}) y_t \right), \tag{17}$$

and

$$\mathbf{x}_\mathbf{t}(\hat{\boldsymbol{\gamma}}, \hat{\mathbf{c}}) = \left[\mathbf{z}, \mathbf{z}_\mathbf{t}' G(\hat{\boldsymbol{\gamma}}, h(\hat{\mathbf{c}}, s_t) \right]. \tag{18}$$

This is possible because if $\boldsymbol{\gamma}$ and c are known and fixed, the GSTAR model is linear in $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$, which can be easy computed via Conditional OLS (COLS). In a such a way, the nonlinear least square minimization problem, otherwise necessary, more demanding in terms of parameters to estimate and not available in closed-form, reduces to a minimization on three parameters, and is solved via a grid search over γ_1, γ_2, c. In our illustrations, both γ_1 and γ_2 are generally chosen between a minimum value of -10 and a maximum of 10 with rate 0.25 in the first three examples the grid for parameter $c_1$is the set of values computed between the 10^{th} and 90^{th} percentile of s_t with rate computed as the difference of the two and divided for an arbitrarily high number (here, 200). Anyway, this is a only gross rule of thumb and does not pretend to constitute a general indication for application to other data.

A.2 Measures of Predictive Accuracy

The out-of-sample predictive properties of the estimated models are investigated via rolling forecast experiment, according to which the series y_t is divided in a "preforecast" period (going from time $\{1 \ldots t\}$) from which the model is estimated and the h-step-ahead forecast are computed and compared with the "test" period, going from time $\{T^s \ldots T\}$ where $T^s = t + h$; this allows to measure $T - T^s - h + 1$ out-of-sample forecasts. Let denote the corresponding realization of the series as y_t, y_T^s and y_T, as well as the corresponding density forecasts as f_t, f_T^s and f_T. Since our interest lies in short-run forecasting we consider $h = \{1, 3, 6, 12\}$. The point predictive performances of the model j are investigated by four different measures: the mean forecast error (MFE), the symmetric mean absolute percentage error (sMAPE), the median relative absolute error (mRAE) and the root mean square forecast error (RMSFE)[13]:

$$MFE_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(y_{t+h} - \hat{y}^j_{t+h|t} \right) \tag{19}$$

$$sMAPE_{j,h} = \frac{100|y_{t+h} - \hat{y}^j_{t+h}|}{0.5(y_{t+h} - \hat{y}^j_{t+h|t})} \tag{20}$$

$$mRAE_{j,h} = \frac{|y_{t+h} - \hat{y}^j_{t+h}|}{|y_{t+h} - \hat{y}^{(1)}_{t+h}|}, \quad \text{with (1) indexing the benchmark model;} \tag{21}$$

$$RMSFE_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(y_{t+h} - \hat{y}^j_{t+h|t} \right)^2 \tag{22}$$

Differently, the literature on aggregation of density forecasts is instead in a development phase, and focuses on the so called *scoring rules* (or *opinion pools*), peculiar functions enabling the forecaster to properly aggregate the set of conditional predictive density as well as more common measures as Mean Square Forecast Error *et similia* do for point forecasts. Despite their dated origins in statistics, as documented by Gneiting and Raftery (2007), scoring rules are becoming increasingly applied by contemporaneous econometric literature only recently; see, *inter alia*, Geweke and

[13]In particular, sMAPE and mRAE are recommended when the series is known to present volatility effects or skewness, two features often associated to nonlinearity; see the discussion in Tashman (2000).

Amisano (2011). In a similar fashion, concerning about density forecasting, four different scoring rules are used for aggregate the $T - T^s - h + 1$ predictive densities produced by the same forecasting exercise:

- the logarithmic score (LogS):

$$LogS_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} log \hat{f}^j_{t+h|t} \tag{23}$$

  corresponding to a Kullback-Liebler distance from the true density; models with higher LogS are preferred.

- The quadratic score, somehow the equivalent of the MSFE in point forecasting, is defined as:

$$QSR_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \sum_{k=i}^{K} (f^j_{t+h|t} - d_{kt})^2, \tag{24}$$

  where $d_{kt} = 1$ if $k = t$ and 0 otherwise; models with lower QSR are preferred.

- The (aggregate) continuous-ranked probability (CRPS) score, equivalent to the sMAPE, is defined as:

$$CRPS_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(|f_{t+h} - \hat{f}^j_{t+h|t}| - 0.5|f_{t+h} - f'_{t+h}| \right), \tag{25}$$

  where f and f' are independent random draws from the predictive density and $f_{t+h|t}$ the observed value; models with lower CRPS are preferred.

- Finally, the quantile score (qS) can be obtained if $f^j_{t+h|t}$ is replaced in (23) by a predictive α-level quantile $q^\alpha_{t+h|t}$ (and the logarithmic function removed); this score is used in risk analysis because it provides information about deviations from the true tail of the distribution.