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Bayesian Credit Rating Assessment

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ABSTRACT

In this contribution we aim at improving ordinal variable selection in the context of causal models. In this regard, we propose an approach that provides a formal inferential tool to compare the explanatory power of each covariate, and, therefore, to select an effective model for classification purposes. Our proposed model is Bayesian nonparametric, and, thus, keeps the amount of model specification to a minimum. We consider the case in which information from the covariates is at the ordinal level. A noticeable instance of this regards the situation in which ordinal variables result from rankings of companies that are to be evaluated according to different macro and micro economic aspects, leading to different ordinal covariates that correspond to different ratings, that entail different magnitudes of the probability of default. For each given covariate, we suggest to partition the statistical units in as many groups as the number of observed levels of the covariate. We then assume individual defaults to be homogeneous within each group and heterogeneous across groups. Our aim is to compare and, therefore, select, the partition structures resulting from the consideration of different explanatory covariates. The metric we choose for variable comparison is the calculation of the posterior probability of each partition. The application of our proposal to a European credit risk database shows that it performs well, leading to a coherent and clear to explain method for variable averaging the estimated default probabilities.

1. BACKGROUND

In this contribution we aim at improving categorical variable selection in the context of causal models. In this regard, we propose an approach that provides a formal inferential tool to compare the explanatory power of each covariate, and, therefore, to select an effective model for predictive purposes. Our proposed model is Bayesian nonparametric, and, thus, keeps the amount of model specification to a minimum.

We consider the case in which information from the covariates is at the ordinal level. A noticeable instance of this regards the situation in which ordinal variables result from rankings of the available statistical units based on specific types of information. As a running example consider the case in which the statistical units represent companies that are to be evaluated according to different macro and micro economic aspects, leading to different ordinal covariates that correspond to different ratings that entail different magnitudes of the probability of default (see e.g. Altman, 1968). For each given covariate, we partition the statistical units in as many groups as the number of observed levels of the covariate. We then assume individual default probabilities to be homogeneous within each group and heterogeneous across groups. Our aim is to compare the partition structures resulting from the consideration of different explanatory covariates. The metric we choose for the comparison is the calculation of the posterior probability of each partition.

Once the grouping scheme is completed, there are two further assumptions to be made, conditionally on the assumed partition. The first one concerns the dependence structure between the individual observations within the same group. The second one concerns the dependence structure between observations for individuals belonging to different groups. A feasible assumption is that, within each group, observations are considered exchangeable. This implies that the observations follow the partial exchangeable scheme proposed by De Finetti (1938).

More formally assume that there are K explanatory covariates of the default event, each characterized by a specific collection of ordinal categories (levels): $j = 1, \ldots, J$. The K covariates are candidate predictors for a binary response variable that represents the default event of each company: $\{Y_i; i = 1, \ldots, n\}$.

Let g_k with $k = 1, \ldots, K$ be a partition of the n observation units induced by the $k - th$ covariate . We consider a hierarchical non parametric approach and we assign the distribution of the random vector of the cumulative distribution function $(F_1, \ldots, F_J)$, assuming that $F_1, \ldots, F_J$ are conditionally independent given a vector of parameters $\theta = (\theta_1, \ldots \theta_J)$ with

$$(F_j|\theta) \sim D(\alpha(\theta_j)) \tag{1.1}$$

where $D(\alpha(\theta_j))$ is a Dirichlet process with parameter $\alpha(\theta_j)$ (see Ferguson, 1973). Furthermore, θ is taken to be a random vector with distribution function H , so that

$$(F_1, \ldots, F_J) \sim \int_{R^J} \prod_{i=1}^{J} D(\alpha(\theta_j))H(d\theta) \tag{1.2}$$

The resulting process is precisely a mixture of products of Dirichlet processes (MPDP), as introduced in the literature by Cifarelli and Regazzini (1978). Some applications of MPDP processes are: Cifarelli (1979), Cifarelli et al. (1981), Consonni (1981), Muliere and Scarsini (1983), Muliere and Petrone (1993), Mira and Petrone (1996), Walker (1999), Carota and Parmigiani (2000), Giudici et al. (2003).

We remark that in our context we assume to be provided with K covariates on which we base the partitions, that represent different ratings of the companies. Such covariates may be obtained from statistically sound predictive models of the default event but typically the data behind the ratings is undisclosed. An example are the financial ratings from the major agencies: Standard & Poors, Fitch, Moody's.

We also remark that the approach underlying our method is also related to that pervading trees models (see for instance, Wu et al. 2007), which also leads to the identification of an optimal partition structure, starting from a different, recursive, modellisation of the binary default variable.

The performance of our methodology will be illustrated by means of a real data set aimed at credit rating assessment of 1000 European companies.

We shall demonstrate that our methodology can be not only coherent from a methodological view point but also useful from a practical side. We also compare the results obtained with our approach with those under a simpler parametric Bayesian analysis.

The paper is organized as follows: in Section 2 we present and discuss our proposed methodology; in Section 3 we consider the computational and applied issues involved in the implementation of the methodology ; finally, Section 4 contains further remarks and discussion.

2. METHODOLOGY

Before describing our proposal, we remark that, to ease the presentation of our methodology, we shall consider as running example the real application in credit rating assessment that will be fully described in Section 3.

In this contribution we propose a methodology to select covariates aimed at estimating the probability of default of companies, using efficiently the provided information typically contained in several distinct databases.

The final aim is to classify companies into groups in a supervised way. Such groups, in order to comply with financial regulations (see e.g. www.bis.org), have to be: homogeneous with regard to the response variable (default), order preserving and stable with regard to horizon time. In this context we are typically provided with databases of various origin, often undisclosed and made of both qualitative and quantitative variables. Our proposal is to build, effective but easy to explain, ordinal rating models integrated by means of non parametric Bayesian analysis.

More precisely, we predict the response variable by averaging the selected variables with the employment of Bayes non parametric modeling. We consider the case in which information from the covariates is at the ordinal level; thus we partition the companies in as many groups as the number of observed levels of the considered covariate. Moreover, once the partitioning scheme is concluded, we are requested to formulate two important assumptions: the dependance structure between the observations within the same group and the dependance structure between observations from different groups. We assume that within each group observations are considered exchangeable (see De Finetti, 1938) and that between groups observations are independent.

Let θ_i be the quantity of interest for $i = 1, \cdots, n$ companies to be rated. We propose to estimate the probability of default of each company, on which the ratings will be based as follows:

$$E(\theta_i \mid \underline{X}, Y) = \sum_{k=1}^{K} E(\theta_j \mid g_k, Y) \cdot p(g_k \mid Y) \tag{2.1}$$

where g_k is a partition induced by a covariate X_k that classifies each unit i into one and only one level j and Y is the observed response variable that assumes two levels: default or not default.

The parameter θ can be either an unknown distribution function (in the non parametric case) or an unknown parameter that fully specifies the distribution function (in the parametric case). Although here we focus on the non parametric case we shall also consider, for the sake of comparison, the parametric case.

Equation (2.1) implies that each covariate induces through the associated partition g_k, j estimates of the probability of default that will be shown to be weighted averages between the prior expectation and the sample mean default.

In a typical credit rating context, many partitions could arise from different covariates corresponding to different databases information. Equation (3) shows how to combine the corresponding estimates of default through the theorem of total probability.

In Equation (2.1) the posterior probability of the partition g_k, $p(g_k|Y)$ is proportional to $p(y|g_k) * p(g_k)$ where $p(y|g_k)$ is the marginal likelihood of g_k and the prior distribution $p(g_k)$ can be set a priori, for example according to the uniform distribution: $p(g_k) \propto \frac{1}{S}$ where S is a constant. A possible choice can be $S = K$.

We now describe how formula 3 can be detailed in the non parametric context. Formally, let $Y_1 \dots Y_n$ be n observations on default events (where $Y_i = 0$ is non default and $Y_i = 1$ is default)

Let $X_1 \dots X_K$ be K covariates and $g_1 \dots g_K$ the associated partitions, according to which the units are clustered into groups in the same number as the measurement levels J_K. Let g_K be one of such partitions with levels $j = 1, \dots, J_K$.

Assume that for the J_k levels of g_k, conditionally on the J_K cumulative distribution functions $F_1, \cdots, F_{J_K}$, independence between observations in different groups holds:

$$P(Y_{1_1} \leq y_{1_1}, \dots, Y_{1_{n1}} \leq y_{1_{n1}}, \dots, Y_{J_{n_1}} \leq y_{J_{n_1}} | F_1, \dots, F_{J_K}) = \prod_{j=1}^{J_K} \prod_{i=1}^{n_j} F_j(y_{j_i}) \tag{2.2}$$

The previous assumptions imply that the observations $Y_{i_{ni}}$, $i = 1, \dots, n_i$ are not independent. The cumulative distribution functions (cdfs) in formula (4) are random quantities and therefore we need to assign them a distribution. We assume that each cdf is distributed according to a Dirichlet process, with parameter $\alpha(\theta_j)$:

$$F_j \sim D(\alpha(\theta_j)) \tag{2.3}$$

We also assume that $\alpha(\theta_j) = MF_0(\theta_j)$ where $F_0(\theta_j)$ is a cdf of a distribution of a known form. Since our aim is to model the default or not default event we take $F_0(\theta_j) \sim Bernoulli(\theta_j)$ that represents the a priori cdf. M is the related measure of precision, a constant number that is defined by the researcher.

In order to avoid a strong dependance of the results on prior specification we take θ_j independent and identically distributed as $Beta(\alpha, \beta)$ with α and β to be specified by the researcher. Note that according to the latter assumption only 3 hyperparamters have to be specified: α and β which express the prior expected default and M that expresses the prior precision of such expectation.

Let $Y_{1_j} \dots Y_{n_j}$ be a sample from a $D(\alpha)$ for the level j of a partition g_k. Thus the posterior distribution of F_j given $Y_{1_j} \dots Y_{n_j}$ can be easily shown to be

$$D(\alpha + \sum_{i=1}^{n_j} y_{i_j}) \tag{2.4}$$

where $\sum_{i=1}^{n_j} y_{i_j}$ is the number of defaults in the j-th level of the considered covariate and n_j is the total number of observations in the same level j. To ease the notation, in the following we indicate $\sum_{i=1}^{n_j} y_{i_j}$ with d_j.

It can then be shown that the minimum squared loss Bayesian optimal rule, the a posteriori mean default, is equal to:

$$E(\theta_j|Y, g_k) = \frac{M}{M+n_j}\left[\frac{\alpha}{\alpha+\beta}\right] + \frac{n_j}{M+n_j}\left[\frac{d_j}{n_j}\right] \tag{2.5}$$

Equation (2.5) derives the estimated probability of default for all companies sharing the same level j of the $k-th$ covariate which, multiplied by the posterior probability of partition g_k in formula (3) will define the final contribution of the covariate k to the probability of default of a single company .

We now move to the calculation of the posterior probability of a partition on which we shall focus the variable selection methodology in this paper.

Let $Y_{1_j} \dots Y_{n_j}$ be a sample from a $D(\alpha)$ for the level j. Antoniak (1973) and Petrone, Raftery (1997) derived the general expression of the marginal likelihood of a Dirichlet process.

Our contribution is the derivation of the specific expression under the assumptions described before that concern ordinal covariates. After some calculations we have obtained that for a generic partition g_k the marginal likelihood contribution of a level j is equal to:

$$p(\underline{y}_j) = \frac{M^{r_j}}{M^{[n_j]}}(d_j - 1)! \left[\frac{\alpha+\beta}{\alpha}\right](nd_j - 1)! \left[\frac{\alpha+\beta}{\beta}\right] \tag{2.6}$$

from which the total marginal likelihood of the partition is equal to:

$$p(\underline{y}|g_k) = \prod_{j=1}^{J} p(\underline{y}_j) \tag{2.7}$$

Remark: The parametric case.

For the sake of comparison we now describe present the derivation of Equation (2.1) in a simpler parametric context.

We now assume that the probability of default of each company is constant and equal to θ_j within the same level j of the covariate. Assuming that θ_j are independent Beta random variables with parameters $(\alpha,\, \beta)$, it can be easily shown that the Bayesian estimate of θ_j is:

$$E(\theta_j \mid Y, g_k) = \frac{\alpha + d_j}{\alpha+\beta+n_j} =$$

$$\left[\frac{\alpha+\beta}{\alpha+\beta+n_j}\right]\frac{\alpha}{\alpha+\beta} + \left[\frac{n_j}{\alpha+\beta+n_j}\right]\frac{d_j}{n_j} \tag{2.8}$$

From Equation (2.8) note the evident analogy with Equation (2.5) with $\alpha+\beta$ playing the role of M in the specification of the prior precision. Indeed the presence of an extra hyperparameter in the non parametric case allows more flexible prior specification that may lead to a more selective variable choice without hampering the specification of the prior expectation .

Concerning the calculation of the posterior probability of each partition in the parametric case it can be shown that the marginal likelihood is equal to:

$$p(y \mid g_k) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right]^J \prod_{j=1}^{J} \frac{\Gamma(\alpha+d_j)\Gamma(\beta+nd_j)}{\Gamma(\alpha+\beta+nd_j)} \tag{2.9}$$

and once again

$$p(g_k \mid Y) \propto p(y \mid g_k)p(g_k) \tag{2.10}$$

is the posterior probability that acts as the covariate weight in Equation (2.1).

3. APPLICATION TO CREDIT RISK MODELING

The financial crisis of 2008 questioned the validity of credit risk models and their practical implications. Within the framework of existing regulatory models, banks have a tendency to uniform their models of risk evaluation and enterprise funding generating a pro-cyclical approach that in the current economic and financial environment highlights and exacerbates the difficult conditions in which the firms operate. Over the last 25 years, technological advances and information sharing have increased the use of credit scoring in almost all forms of loan origination (Altman and Saunders, 1997, Altman, 1968). Theoretical studies have demonstrated the importance of information sharing in mitigating the problems of adverse selection (Jappelli and Pagano, 1993) and moral hazard (Padilla and Pagano, 2000). However, the high dimensional data available from public financial statements make credit analysis difficult, and the problems are exacerbated by the necessity to account jointly for qualitative and quantitative data. In order to improve empirical results, and obtain credit ratings that are more predictive and less procyclical, research is needed in the area of variable selection, especially for ordinal variables, preliminary to the inclusion in a full Bayesian averaging perspective. Our proposal in last section aims to deal with the above model in a way that is at the same time statistically sound and reasonably simple to understand.

In order to evaluate the performance of our proposed model, we were provided from with a database containing a collection of ordinal rating variables on a set of companies from different industries. Specifically, the dataset is made up of 1000 companies and 13 variables. A response variable describing the occurrence of the default or not default event; the remaining 12 on an ordinal measurement. These 12 variables can be divided in two subsets: the first containing 3 independent external rating evaluations and the second containing 9 rating items from the internal rating questionnaire. The three external ratings, all measured on a ordinal scale of 9 levels, are based on three different databases : 'Ai' describing the banking transactions of the involved companies,'Dir' containing macro-economic scenarios and 'Cebi' including balance sheets information.

On the internal rating side we have 9 ordinal variables characterized by a 4 levels measurement, those variables give information either on the historical relation between the company and the bank (if there exists) or on the management and structure of the company itself. In particular: question Q1 concerns 'Competitive position of the company'; Q2 is 'Ability to change the management board without financial consequences'; Q3 is 'Payment of suppliers'; Q4 is 'Number of clients which the 50% of sales refers to'; Q5 is 'Payment from clients'; Q6 'Trend of the demand for the goods produced by the company'; Q7 'Historical relation with the bank'; Q8 'Financial ability of the board to face adverse economic conditions'; Q9 'Professional experience of the management'. The order of the scales for all 9 covariates is related with the risk level: higher levels of each covariate means higher default risk.

We now proceed with variable selection according to both the non parametric and parametric models described in Section 2.

First of all we address the issue of unequal number of levels of the 12 covariates measurement scales. To avoid such difference affecting the results in terms of variable selection we need to appropriately 'standardize' the measurement scales. We propose to reduce the granularity of the scales to that of the response variable, that is to dichotomize each covariate into two classes that, doing so, respect the binary nature of the default variable.

Since we do not have a priori information on how to binarize and we look for a formally coherent choice that can avoid rules of thumbs. The Bayesian framework proposed by us is suitable for this task by selecting, not only the most relevant variables, but also the best dichotomization by maximizing the corresponding posterior probabilities.

We thus compute the marginal likelihood for each covariate by varying the dichotomization threshold. Such procedure allows us to choose the covariate binarization with the highest posterior probability.

In Table I we report the posterior marginal probabilities of the 9 internal rating items (columns) crossed with the dichotomization thresholds (rows). We underline that, the covariates Q3, Q4, Q5 and Q7 have been treated with particular attention since one of the modalities of the measurement scale represents the 'not available' state. To avoid such missing information to bias the results we removed it, renormalizing the observed proportions accordingly.

Table1: Non parametric Posterior probabilities varying the dichotomization of internal ratings (in bold best configuration)

Dichot.	Q1	Q2	Q6	Q8	Q9
1 vs 2:4	0.04	0.29	0.03	0.06	0.04
1:2 vs 3:4	**0.72**	**0.68**	0.36	**0.93**	**0.94**
1:3 vs 4	0.24	0.03	**0.61**	0.01	0.03
Dichot.	Q3	Q4	Q5	Q7	
1 vs 2:3	**0.76**	**0.72**	0.03	**0.91**	
1:2 vs 3	0.24	0.28	**0.97**	0.09	

From Table I it turns out that the best dichotomization for most 4 level scales is 1:2 vs 3:4, with the exception of variable Q6 for which the highest risk level (number 4) should be dichotomized against 1:3. For the three level scales resulting from the dropping out of the not available level the best dichotomization is typically 1 vs 2:3, with the exception of covariate Q5. For the sake of comparison table 2 shows analogous results for the parametric models.

Table II: Parametric Posterior probabilities varying the dichotomization of internal ratings (in bold best configuration)

Dichot.	Q1	Q2	Q6	Q8	Q9
1 vs 2:4	0.28	0.33	0.30	0.39	0.33
1:2 vs 3:4	**0.45**	**0.43**	**0.37**	**0.46**	**0.36**
1:3 vs 4	0.27	0.24	0.32	0.25	0.30
Dichot.	Q3	Q4	Q5	Q7	
1 vs 2:3	**0.59**	**0.50**	0.06	**0.73**	
1:2 vs 3	0.41	0.50	**0.94**	0.27	

From Table II note that the selected dichotomizations are the same as the non parametric case, with the exception of Q6 that becomes in line with the others.

In Table III we report the posterior marginal probabilities of the 3 external rating items (columns) crossed with the dichotomization thresholds (rows).

Table III: Non Parametric Posterior probabilities varying the dichotomization of external ratings

Dichotomization	CEBI	AI	Dir
1-2:9	0.00	0.00	**0.54**
1:2-3:9	0.00	0.00	0.08
1:3-4:9	0.05	0.01	0.07
1:4-5:9	0.07	0.01	0.05
1:5-6:9	0.21	0.02	0.04
1:6-7:9	**0.48**	0.08	0.06
1:7-8:9	0.03	**0.75**	0.06
1:8-9	0.16	0.12	0.11

From Table III note that the dichotomization thresholds vary and polarize the distributions. For the 'Cebi' and 'Ai' the best dichotomizations split high risk levels versus the others, for 'Dir' is the opposite.

For the sake of comparison Table IV shows analogous results for the parametric models.

Table IV: Parametric Posterior probabilities varying the dichotomization of external ratings

Dichotomization	Cebi	Ai	Dir
1 vs 2:9	0.09	0.05	0.12
1:2 vs 3:9	0.10	0.07	0.12
1:3 vs 4:9	0.10	0.11	0.13
1:4 vs 5:9	0.14	0.14	0.12
1:5 vs 6:9	0.18	0.16	0.12
1:6 vs 7:9	**0.19**	**0.20**	**0.13**
1:7 vs 8:9	0.09	0.19	0.13
1:8 vs 9	0.09	0.08	0.12

Table IV shows results that are more stable since the best split is always 1:6 versus 7:9.

We finally move to the implementation of variable selection. Because of their different nature, we have performed variable selection for the internal and the external variables separately. Concerning variable selection, among the 9 internal variables the posterior probabilities are contained in Table V and VI.

Table V: Parametric Posterior probabilities for the internal ratings (in bold best configuration)

Q1	Q2	Q6	Q8	Q9
0.00	0.01	0.01	0.01	0.01
Q3	Q4	Q5	Q7	
0.84	0.00	0.12	0.00	

Table VI: Non Parametric Posterior probabilities for the internal ratings (in bold best configuration)

Q1	Q2	Q6	Q8	Q9
0.01	0.02	0.00	0.00	0.01
Q3	Q4	Q5	Q7	
0.79	0.06	0.10	0.00	

Based on both the best variable is by far Q3 that receives a posterior probability of 0.84 and 0.79 respectively.

Table VII: Non Parametric Posterior probabilities varying the dichotomization of external ratings

Cebi	Ai	Dir
0.01	0.95	0.04

Table VIII: Parametric Posterior probabilities varying the dichotomization of external ratings

Cebi	Ai	Dir
0.04	0.89	0.07

On the other hand, in Table VII and VIII we report the posterior probabilities of the 3 external rating variables.

Once again the non parametric and the parametric methodologies are consistent: the best partition is that associated with 'AI' followed by 'Dir' and 'Cebi'. However the reader can note that the non parametric approach tends to amplify the differences even if the ranking is not changed.

Finally, we want to produce a final weights configuration, useful for companies classification purposes. In order to avoid the overestimating of the 9 internal covariates, we decided to combine only the variable with the highest explanatory power (i.e. Q3) with the external ratings covariates and obtain in Table IX the final weights of the most relevant variables that can be eventually plugged into Equation (2.1).

Table IX: Posterior probabilities of the best configurations

Parametric	Q3	Cebi	Ai	Dir
	0.5000	0.0976	0.2173	0.1849

From Table IX note that the internal rating variable has the highest importance. This is a very important result which suggests that financial institutions should in their credit rating models take into account not only external ratings (such ad those coming from the rating agencies) but also those coming from direct and personal knowledge of their customer companies.

4. CONCLUSIONS

Our model can be seen as a very useful integrated method within credit risk assessment, which typically uses multiple sources of information: balance sheets, assessment questionnaires, bank account flows, rating from credit agencies. In credit rating assessment a structured approach is needed, that fully employs the potential of Bayesian modelling, a natural way to merge different information.

We address the problem of integrating several credit scores, by homogenizing the variables level measurement and importance. In particular, we binarize the ordinal variables, typically arising from assessment questionnaires and ratings, to be matched with the target variable (default or not default). We have also proposed a novel Bayesian variable averaging procedure that leads to produce efficient estimation of probabilities of default and relative rating classes when dealing with several variables of different nature on a set of companies. Such variables averaging approach will also be able to select the most important covariates and the relative best binarization in terms of predicting the target event: default or non default.

Future research development include a thorough predictive performance tests of the model and the extension to longitudinal data, when they will be available. In view of such extensions we may need a methodology able to summarize internal rating variables into summary indicators. One possibility is to follow what recently proposed by Cerchiello et al (2010) that suggests to employ stochastic dominance and quantile-based indicators. A further need in modelling of microeconomic credit risk data is to take into account interdependencies between risk variables, and their causal factors; one possibility is to employ bayesian network modelling as suggested in Cornalba and Giudici (2004) or Bonafede et al.(2007) or, alternatively, Bayesian graphical models as suggested in Giudici (2001).

Finally we remark that what proposed here can be applied to other assessment context such as quality and reputational risk as introduced in Cerchiello (2011).

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