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**Open-economy Inflation Targeting Policies  
and Forecast Accuracy**

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# Open-economy Inflation Targeting Policies and Forecast Accuracy

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## Abstract

Forecast accuracy in macroeconomics is based on statistical techniques for extrapolating time series. This paper takes a new theoretical route studying the relation between forecast accuracy of macroeconomic variables and alternative monetary policies. Considering optimal policy with model-parameter uncertainty in a small open-economy, the paper shows that Domestic Inflation Targeting (DIT) leads to more forecast accuracy than Consumer Price index Inflation Targeting (CPIIT). Furthermore, forecast accuracy and policy aggressiveness turn out to be inversely related, and the trade-off is more severe under CPIIT. These results are obtained in a New-Keynesian model measuring forecast accuracy by the volatility of simulated fan-charts.

JEL Classification: E52, E58, F41.

Key Words: Multiplicative uncertainty; Markov jump linear quadratic systems; small open-economy; optimal monetary policy; inflation index.

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# 1 Introduction

Forecasting the evolution of the economy is a key monetary policy issue whose relevance became apparent with the advent of Inflation Targeting. This monetary regime highlighted how long and variable lags in the transmission of the monetary stimulus, along with the exposure to exogenous disturbances, require an operating procedure based on *distribution forecast targeting* for which what matters is the whole expected distribution forecast rather than its mean value.

Not less important, central bank transparency about internal forecasts characterizes at various levels inflation targeting central banks nowadays. Thus, proficiency at forecasting also helps to shape the expectations of the private sector enhancing in this way monetary policy effectiveness. This is the so called expectations channel. Its relevance in the monetary policy transmission mechanism is well captured by the consolidated view that successful monetary policy is, mainly, the management of the market expectations, as Woodford (2001) initially put it.

Forecasting the evolution of the economy, however, is challenging due to the well known problem posed by limited information on the state and model of the economy, current shocks and future contingencies. Motivated by the central banks' need to improve the quality of the economic outlook, this paper investigates whether forecasts accuracy can depend on the central bank type of Inflation Targeting monetary policy. Thus, considering a New Keynesian small open economy, this exercise asks to what extent, if any, forecasts accuracy depend on the choice between CPI and Domestic Inflation Targeting (henceforth CPI IT and DIT) and finds that this choice does matter. Indeed, the first result is that, under optimal monetary policy, the stabilization of CPI inflation tends to be inversely related to the accuracy of the distribution forecasts of the other macrovariables. Thus, the first contribution of the current work lies in unveiling DIT as the policy that performs best at forecast accuracy for most of the macrovariables. The analysis also shows the presence of a trade-off between forecast accuracy and aggressiveness of the optimal monetary policy, the latter depending on the central bank preferences on smoothing the interest rate. Interestingly, under CPI IT this trade-off is more severe than under DIT. This finding is relevant as it suggests that if external circumstances require a change in gear for policy aggressiveness, then the cost in terms of less forecast accuracy is lower with DIT.

The intuition for these findings is based on the combined action of two factors: the *level of policy activism* implied by the choice of the inflation targeting policies, and the *consideration*

*of model parameter uncertainty* on the part of the central bank. Since under CPI IT there is more policy activism than under DIT, when the central bank decides the optimal policy and takes into account model parameter uncertainty a more active policy results in more volatility for most of the macrovariables.

Investigating the relation between the quality of the forecasts and the inflation targeting policies requires, first and foremost, a measure of forecast accuracy. In the current theoretical exercise, this measure is generated by simulating distribution forecasts in response to exogenous shocks and then computing the volatility of these distribution forecasts. Hence, with a proper measure of forecast accuracy associated with alternative targeting policies we can green light the policy race.

As to optimal policy, a standard way to find it consists of modeling central bank and private sector behaviors with a quadratic loss function and linear aggregate demand and supply, respectively. This approach, in presence of additive exogenous disturbances, leads to the well known Certainty Equivalence result: the same optimal policy with or without shocks. Thus the model would generate *mean forecasts* for each macrovariables in response to a shock, i.e. impulse response function, rather than *distribution forecasts*, i.e. impulse response distribution forecasts.

To avoid Certainty Equivalence and therefore obtain useful distribution forecasts, we relax the strong assumption usually held in the literature that central banks know with certainty the model of the economy. Thus, when an exogenous shock hits, several possible expected paths of the economy are possible, which result in a distribution forecast for each macrovariable. To consider model parameter uncertainty, which has nature of *multiplicative* uncertainty, along with exogenous shocks, which instead have nature of *additive* uncertainty, the modeling strategy follows the Svensson and Williams (2007) approach based on Markov jump-linear-quadratic systems.

Model uncertainty appears in the optimal policy literature with the seminal work of Brainard (1967) suggesting that considering model parameter uncertainty attenuates the policy action. This conclusion is qualified by Craine (1979) and Söderström (2002) who respectively showed in a backward-looking one- and two-equation model that the policy response may attenuate or increase depending on the type of parameters surrounded by uncertainty<sup>1</sup>. The current work differs from these previous papers and the subsequent literature because does not study

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<sup>1</sup>These conclusions are also extended to forward-looking models by Moessner (2005), Kimura and Kurozumi (2007), and Flamini and Milas (2011).

how model uncertainty affects the coefficients of the optimal policy. What the paper focuses on is how the consideration of model parameter uncertainty makes the comparison between alternative inflation targeting policies relevant in terms of forecast accuracy.

The paper is organized as follows. Section 2 presents the model and its calibration. Model simulations under the alternative inflation targeting policies are reported and discussed in Section 3 where it is also analyzed the role played by model parameter uncertainty in the policy assessment. Conclusions are in Section 4.

## 2 The model

The model draws on Flamini (2007) and assumes that in the design of the optimal monetary policy central banks have limited information on the behaviour of the private sector<sup>2</sup>.

### 2.1 The household

The economy is populated by a continuum of consumers/producers indexed by  $j \in [0, 1]$  sharing the same preferences and living forever. The representative household seeks to maximize the expected value of an intertemporal utility of the form

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau U(C_{t+\tau}, \check{C}_{t+\tau-1}), \quad (1)$$

where  $\delta$  is the intertemporal discount factor,  $C_t$  is total consumption of household  $j$ , and  $\check{C}_t$  is the total aggregate consumption. Preferences over total consumption feature habit formation *à la* Abel (1990) captured by the following instantaneous utility function

$$U(C_{t+\tau}, \check{C}_{t+\tau-1}) = \frac{(C_{t+\tau}/\check{C}_{t+\tau-1})^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}, \quad (2)$$

where  $\sigma > 0$  is the intertemporal elasticity of substitution and  $\iota \geq 0$  captures habit persistence. Habit persistence determines the degree of backward and forward lookingness of the household, and therefore the degree of persistence in the aggregate demand. The previous literature offered a wide range of estimations for habit persistence to which corresponds a wide range of aggregate demands. This range spans from purely backward looking aggregate demands, where a change in the previous period output gap leads to the same change in the current period output gap, to

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<sup>2</sup>This section reports a concise description of the model in order to allow a clear presentation of how *model uncertainty* affects the expected dynamics of the economy. Details on the derivation of the structural relations can be found in Flamini (2007).

completely forward looking aggregate demands, where the previous period output gap does not affect the current period output gap<sup>3</sup>. Given the variety of proposed values for habit persistence, this work assumes that the central bank does not choose a specific value for this parameter but a range. In other words, the central bank is uncertain on the amount of persistence in the aggregate demand.

Back to the model, total consumption,  $C_t$ , is a Cobb-Douglas function of domestic good consumption,  $C_t^d$ , and import good consumption,  $C_t^i$ ,

$$C_t \equiv C_t^{d(1-w)} C_t^{iw}, \quad (3)$$

where  $w$  determines the steady state share of imported goods in total consumption and  $C_t^d, C_t^i$  are Dixit-Stiglitz aggregates of continuum of differentiated domestic goods and import goods (henceforth indexed with  $d$  and  $i$  respectively),

$$C_t^h = \left[ \int \left( C_t^h(j) \right)^{1-\frac{1}{\vartheta}} dj \right]^{\frac{1}{1-\vartheta}}, \quad h = d, i,$$

where  $\vartheta > 1$  is the elasticity of substitution between any two differentiated goods and, for the sake of simplicity, is the same in both sectors<sup>4</sup>. Finally,  $P^c$  is the overall Dixit-Stiglitz price index for the minimum cost of a unit of  $C_t$  and is given by

$$P_t^c = \frac{P_t^{iw} P_t^{d(1-w)}}{w^w (1-w)^{(1-w)}}, \quad (4)$$

with  $P^d, P^i$  denoting, respectively, the Dixit-Stiglitz price index for goods produced in the domestic and import sector.

Assuming a no-Ponzi schemes condition, utility maximization subject to the budget constraint and the limit on borrowing gives the Euler equation and the Uncovered Interest Parity, which in terms of log deviations from steady state values are, respectively

$$c_t = \beta c_{t-1} + (1-\beta) c_{t+1|t} - (1-\beta) \sigma \left( i_t - \pi_{t+1|t}^c \right), \quad \beta \equiv \frac{\iota_t (1-\sigma)}{1 + \iota_t (1-\sigma)} < 1, \quad (5)$$

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<sup>3</sup>For a review of the previous literature on the calibration of habits formation see Leith, Moldovan and Rossi (2009).

<sup>4</sup>Following Corsetti and Pesenti (2004), the intratemporal elasticity of substitution between domestic and import goods is set equal to one. This assumption ensures the stationarity of the model.

$$i_t - i_t^* = s_{t+1|t} - s_t + v_t, \quad (6)$$

where for any variable  $x$ , the expression  $x_{t+\tau|t}$  stands for the rational expectation of that variable in period  $t + \tau$  conditional on the information available in period  $t$  and, by means of a log-linearization, the variables  $c_t$ ,  $\pi_t^c$ ,  $i_t$ ,  $i_t^*$ ,  $(s_{t+1|t} - s_t)$  and  $v_t$  are log-deviations from their respective constant steady state values; finally,  $c_t$  denotes total aggregate consumption, obtained considering that in equilibrium total consumption for agent  $j$  is equal to total aggregate consumption, i.e.  $C_t = \check{C}_t$ ,  $\pi_t^c$  denotes CPI inflation (measured as the log deviation of gross CPI inflation from the constant CPI inflation target), and  $v_t$  is a risk premium shock added to capture financial market volatility and it is modeled with a stationary univariate AR(1) process

$$v_{t+1} = \gamma_v v_t + \xi_{t+1}^v.$$

### 2.1.1 Domestic consumption of goods produced in the domestic sector

Preferences captured by equation (3) imply that the (log deviation of the) domestic demand for goods produced in the domestic sector,  $c_t^d$ , is given by

$$c_t^d = c_t - \left( p_t^d - p_t^c \right),$$

which, considering the (log-linearized version of the) price index equation (4), can be rewritten as

$$c_t^d = c_t + wq_t, \quad (7)$$

where  $q_t \equiv p_t^i - p_t^d$  is the (log-deviation of the) real exchange rate.

Then, solving equation (5) for  $c_t$  and combining it with equation (7) I obtain

$$c_t^d = -\sigma (1 - F_1 L)^{-1} \rho_t - \sigma (1 - F_1 L)^{-1} wq_t + wq_t, \quad (8)$$

where  $F_1 < 1$  is the smaller root of the characteristic polynomial of equation (5) and

$$\rho_t \equiv \sum_{\tau=0}^{\infty} \left( i_{t+\tau|t} - \pi_{t+\tau+1|t}^d \right) \quad (9)$$

can be interpreted as the long real interest rate.

### 2.1.2 Aggregate demand for goods produced in the domestic sector

Total aggregate demand for the good produced in the domestic sector is

$$\widehat{Y}_t^d = C_t^d + Y_t^{d,d} + Y_t^{d,i} + C_t^{*d}, \quad (10)$$

where  $Y_t^{d,d}$ ,  $Y_t^{d,i}$  and  $C_t^{*d}$  denote the quantity of the (composite) domestic good which is used as an input in the domestic sector, as an input in the import sector and which is demanded by the foreign sector, respectively.

While both sectors feature a continuum of unit mass of firms, indexed by  $j$ , that produce differentiated goods  $Y_t^d(j)$  and  $Y_t^i(j)$  in the domestic and import sector respectively, the two sectors differ for the input used: the domestic sector uses a composite input consisting of the domestic (composite) good itself and the (composite) import good provided by the import sector; the import sector uses a composite input consisting of the foreign good  $Y_t^*$  and the domestic (composite good). Furthermore, to capture the real-world feature that production inputs tend to be rigid at business cycle frequency, sectors are assumed to use a Leontief technology. Thus, the production functions in the domestic and import sector are given respectively by

$$Y_t^d(j) = f \left[ A_t^d \min \left\{ \frac{Y_t^{d,d}}{1-\mu}, \frac{Y_t^{i,d}}{\mu} \right\} \right], \quad Y_t^i(j) = f \left[ A_t^i \min \left\{ \frac{Y_t^*}{1-\mu^i}, \frac{Y_t^{d,i}}{\mu^i} \right\} \right], \quad \mu, \mu^i \in [0, 1], \quad (11)$$

where  $f$  is an increasing, concave, isoelastic function,  $A_t$  is an exogenous (sector specific) economy-wide productivity parameter,  $(1-\mu)$  and  $\mu$  denote, respectively, the shares of the domestic good and import good in the composite input required to produce the differentiated domestic good  $j$ , and  $(1-\mu^i)$  and  $\mu^i$  denote, respectively, the shares of the foreign good and domestic good in the composite input required to provide the differentiated import good  $j$ . Focusing on  $\mu^i$ , it is worth of note that when this parameter is positive a change of the exchange rate does not fully reflect in a change of the import goods price as the composite input consists also of the domestic good. In this case the exchange rate pass-through turns out to be *incomplete*. It is well known that the exchange rate pass-through can be quite variable over time due to numerous factors playing a role in its determination. To model pass-through uncertainty, the parameter  $\mu^i$  is assumed to be uncertain<sup>5</sup>. Returning to the description of the technology,

<sup>5</sup>Campa and Goldberg (2006 and 2005) argue that changes in pass-through can be driven by changes in the use of imported inputs or in the composition of a country's import basket when the component products have distinct pass-through elasticities. Furthermore, various authors (Devereux and Engel 2001, Devereux, Engel and Storgaard 2004, and Devereux and Yetman 2008) link the pass-through variability to changes in monetary stability and the persistence of exogenous shocks, and Bacchetta and van Wincoop (2005) to changes in the market share and in

equation (11) implies that the quantities of the (composite) domestic good used as an input in the domestic and import sector are

$$Y_t^{d,d} = \frac{1}{A_t^d} (1 - \mu) f^{-1} \left( \widehat{Y}_t^d \right), \quad Y_t^{d,i} = \frac{1}{A_t^i} \mu^i f^{-1} \left( \widehat{Y}_t^i \right), \quad (12)$$

where  $\widehat{Y}_t^i$  denotes the demand of the import good. Finally, log-linearizing equation (10) around the steady state values yields

$$\widehat{y}_t^d = \kappa_1 (\mu^i) c_t^d + \kappa_2 (\mu^i) \widehat{y}_t^i + \kappa_3 (\mu^i) c_t^{*d}, \quad (13)$$

where  $\kappa_1' (\mu^i)$ ,  $\kappa_3' (\mu^i) < 0$  and  $\kappa_2' (\mu^i) > 0$ .

Next, the output-gap in sector  $h = d, i$  is defined as

$$y_t^h \equiv \widehat{y}_t^h - y_t^{h,n},$$

where  $y_t^{h,n}$  denotes the log deviation of the natural output in sector  $h$  from its steady state value. As in Svensson (2000), both  $y_t^{h,n}$  and  $c_t^{*d}$  are exogenous and follow, respectively

$$y_{t+1}^{h,n} = \gamma_y^{h,n} y_t^{h,n} + \eta_{t+1}^{h,n}, \quad 0 \leq \gamma_y^{h,n} < 1, \quad h = d, i, \quad (14)$$

where  $\eta_{t+1}^{h,n}$  is a serially uncorrelated zero-mean shock to the natural output level (a productivity shock), and

$$c_t^{*d} = \overline{\beta}_y^* y_t^* + \theta^* w^* q_t, \quad (15)$$

where  $\theta^*$  and  $w^*$  denote, respectively, the foreign atemporal elasticity of substitution between domestic and foreign goods and the share of domestic goods in foreign consumption. Finally, in line with the central banks' view of the approximate one-period lag necessary to affect aggregate demand, consumption decisions are assumed to be predetermined one period in advance. Accordingly, repeating the same derivation with preferences maximized on the basis of one period ahead information results in the aggregate demand in the domestic sector. This relation, expressed in terms of the output-gap, is given by

$$y_{t+1}^d = \beta_y y_t^d - \beta_\rho \rho_{t+1|t} + \beta_q q_{t+1|t} - \beta_{q-1} q_t + \beta_{y^*} y_t^* + \beta_{y^n} y_t^{d,n} + \eta_{t+1}^d - \eta_{t+1}^{d,n}, \quad (16)$$

the degree of differentiation of the exporting country goods.

where  $\eta_{t+1}^d$  is a serially uncorrelated zero-mean demand shock. In (16) all the coefficients are positive and functions of the structural parameters of the model. It is worth noting that, due to the uncertainty on habit persistence, it turns out that, for any period  $t$ , the coefficients for the previous period output gap, real exchange rate, foreign output, and natural output in the domestic sector,  $\beta_y, \beta_{q-1}, \beta_{y^*}, \beta_{y^n}$  respectively, are uncertain.

### 2.1.3 Aggregate demand of goods produced in the import sector

Aggregate demand for import goods is given by

$$\widehat{Y}_t^i = C_t^i + Y_t^{i,d} \quad (17)$$

where  $Y_t^{i,d}$  denotes the amount of the import good used as an input in the domestic sector.

Log-linearizing (17) around the steady state results in

$$\widehat{y}_t^i = (1 - \widetilde{\kappa}) c_t^i + \widetilde{\kappa} \widehat{y}_t^d. \quad (18)$$

Finally, the same assumptions used to derive the aggregate demand for the domestic sector goods yield

$$y_{t+1}^i = \beta_y y_t^i - \beta_\rho \rho_{t+1|t} - \beta_q^i q_{t+1|t} + \beta_{q-1}^i q_t + \beta_{y^*}^i y_t^* + \beta_{y^n}^i y_t^{i,n} + \eta_{t+1}^i - \eta_{t+1}^{i,n}, \quad (19)$$

where all the coefficients are positive and depend on the structural parameters of the model,  $\eta_{t+1}^i$  is a serially uncorrelated zero-mean demand shock, and the coefficients  $\beta_y, \beta_{q-1}^i, \beta_{y^*}^i, \beta_{y^n}^i$  are uncertain.

### 2.1.4 Aggregate supply in the domestic sector

We now assume that firm  $j$  takes

$$Y_t^d(j) = \widehat{Y}_t^d \left( \frac{P_t^d(j)}{P_t^d} \right)^{-\vartheta}$$

as the demand for its own variety, where  $P_t^d(j)$  is the nominal price for variety  $j$ . Since the composite input is a convex combination of both aggregates of domestic and import goods, as shown by equation (11), it follows that the input price is  $W_t \equiv (1 - \mu) P_t^d + \mu P_t^i$ . Furthermore, adopting the Calvo (1983) staggered price scheme, the firm chooses in any period the new

price with probability  $(1 - \alpha)$  or keeps the previous period price indexed to past inflation with probability  $\alpha$ . The parameter  $\alpha$  determines the degree of price stickiness and exerts a major impact on the slope of the Phillips curve, that is the response of inflation to fluctuations in resource utilization. This relation seemed to have varied in the last two decades possibly due to an anchoring of inflation expectations via better monetary policy (Mishkin 2007, Boivin and Giannoni 2006, and Roberts 2006), or due to changes in the price-setting behaviour dependent on the level and variability of inflation (among the others, Cogley and Sbordone 2005 and Rubio-Ramirez and Villaverde 2007). To account for this uncertainty on the slope of the Phillips curve, the parameter  $\alpha$  is assumed to be uncertain. Finally, we assume that when the firm can choose the optimal price, it chooses it two periods in advance. This assumption is motivated by the fact that domestic sector firms take both production and retailing decisions. The implication is that monetary policy needs a two-period lag to affect domestic inflation. This is in line with the central banks' experience of an approximate two-period lag for monetary policy to have the highest impact on inflation. Recalling that all the varieties are produced with the same technology, there is a unique input requirement function for each  $j$  given by  $\frac{1}{A_t^d} f^{-1} [Y_t^d(j)]$  and the variable cost of producing the quantity  $Y_t^d(j)$  is  $W_t \frac{1}{A_t^d} f^{-1} [Y_t^d(j)]$ . It follows that the decision problem for firm  $j$  at time  $t$  is

$$\max_{\tilde{P}_{t+2}^d} E_t \sum_{\tau=0}^{\infty} \alpha^\tau \delta^\tau \tilde{\lambda}_{t+\tau+2}^d \left\{ \frac{\tilde{P}_{t+2}^d \left( \frac{P_{t+\tau+1}^d}{P_{t+1}^d} \right)^\zeta}{P_{t+2+\tau}^d} \hat{Y}_{t+\tau+2}^d \left[ \frac{\tilde{P}_{t+2}^d \left( \frac{P_{t+\tau+1}^d}{P_{t+1}^d} \right)^\zeta}{P_{t+2+\tau}^d} \right]^{-\vartheta}}{f^{-1} \left[ \hat{Y}_{t+\tau+2}^d \left( \frac{\tilde{P}_{t+2}^d \left( \frac{P_{t+\tau+1}^d}{P_{t+1}^d} \right)^\zeta}{P_{t+2+\tau}^d} \right)^{-\vartheta} \right]} \right\}, \quad (20)$$

where  $\tilde{\lambda}_t^d$ ,  $\tilde{P}_{t+2}^d$  and  $\zeta$  denote, respectively, the marginal utility of domestic goods, the new price chosen in period  $t$  for period  $t+2$  and the degree of indexation to the previous period inflation rate<sup>6</sup>. Following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003), the

<sup>6</sup>Recalling that consumption decisions are predetermined one period in advance, the marginal utility of domestic goods  $\tilde{\lambda}_t^d$  is obtained by the following first-order condition with respect to  $C_{t+1}^d$

$$E_t U_d \left( C_{t+1}^d, C_{t+1}^i \right) = E_t \left[ \lambda_{t+1} P_{t+1}^d \right] \equiv E_t \tilde{\lambda}_{t+1}^d,$$

parameter  $\zeta$  introduces inflation inertia in the Calvo model of pricesetting. Empirical evidence on  $\zeta$  is characterized by contrasting results as reported by Kimura and Kurozumi (2007). It is therefore difficult to pin down a value for  $\zeta$  and the paper proceeds by assuming that this parameter belongs to the set of the uncertain parameters.

Finally, following Svensson (2000), I set  $\delta = 1$  to ensure the natural-rate hypothesis and assuming that the purchasing power parity holds in the long run, the log-linearized version of the Phillips curve for the domestic sector turns out to be

$$\pi_{t+2}^d = \frac{1}{1+\zeta} \left[ \zeta \pi_{t+1}^d + \pi_{t+3|t}^d + \frac{(1-\alpha)^2}{\alpha(1+\omega\vartheta)} (\omega y_{t+2|t}^d + \mu q_{t+2|t}) \right] + \varepsilon_{t+2} \quad (21)$$

$$= \phi_\pi \pi_{t+1}^d + (1-\phi_\pi) \pi_{t+3|t}^d + \phi_y^d y_{t+2|t}^d + \phi_q^d q_{t+2|t} + \varepsilon_{t+2}, \quad (22)$$

where  $\omega$  in (21) is the output elasticity of the marginal input requirement function and  $\varepsilon_{t+2}$  is a zero-mean i.i.d. cost-push shock. In (22) all the implicitly defined coefficients are positive and  $\phi_y^d$  and  $\phi_q^d$  are uncertain due to the uncertainty on  $\alpha$  and  $\zeta$ .

### 2.1.5 Aggregate supply in the import sector

In the import sector, the input is a convex combination of the aggregate of domestic goods and of the foreign good, with price  $P_t^* S_t$ , where  $P_t^*$  is the price in foreign currency of the foreign good. It follows that the price of the composite input is  $F_t \equiv \mu^i P_t^d + (1-\mu^i) P_t^* S_t$ .

Now, relaxing the assumption that pricing decisions are predetermined and keeping all the remaining assumptions used to derive the Phillips curve in the domestic sector results in

$$\pi_t^i = \frac{1}{1+\zeta} \left[ \zeta \pi_{t-1}^i + \pi_{t+1|t}^i + \frac{(1-\alpha^i)^2}{\alpha^i(1+\omega\vartheta)} (\omega y_t^i + q_t^i) \right] \quad (23)$$

$$= \phi_\pi \pi_{t-1}^i + (1-\phi_\pi) \pi_{t+1|t}^i + \phi_y^i y_t^i + \phi_q^i q_t^i, \quad (24)$$

where  $\alpha^i$  is the probability of not updating optimally the price in the import sector and is assumed to be uncertain,  $q_t^i$  denotes (the log deviation of) the price of the composite input in the import sector expressed in terms of the import goods price,  $p_t^i$ , and is defined as

$$q_t^i \equiv (1-\mu^i) (s_t + p_t^*) + \mu^i p_t^d - p_t^i, \quad (25)$$

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where  $\lambda_t$  is the marginal utility of nominal income in period  $t$ .

where  $p_t^*$  is the (log) foreign price level. Relaxing the assumption of predetermined pricing decisions is motivated by the fact that the import sector only acts as a retailer for the foreign goods and, in practice, retailers do not set their price before they take effect as much as producers do. It is worthy of note that while  $\mu^i$  determines the degree of completeness of the pass-through as discussed before,  $\alpha^i$  determines the speed of the pass-through. Hence, uncertainty on  $\alpha^i$  and  $\mu^i$  captures two dimensions of the uncertainty on the exchange rate pass-through.

## 2.2 CPI inflation and the uncovered interest parity

CPI-inflation,  $\pi_t^c$ , is given by

$$\pi_t^c = (1 - w) \pi_t^d + w \pi_t^i, \quad (26)$$

where  $w$  is the steady state share of imported goods in total consumption and determines the degree of openness of the economy. In order to eliminate the non-stationary nominal exchange rate, it is convenient to express the Uncovered Interest Parity in terms of  $q_t^i$  obtaining

$$q_{t+1|t}^i - q_t^i = (1 - \mu^i) r_t - (1 - \mu^i) (i_t^* - \pi_{t+1|t}^*) - (\pi_{t+1|t}^i - \pi_{t+1|t}^d) - (1 - \mu^i) v_t, \quad (27)$$

where  $r_t$  is the short term real interest rate defined as  $r_t \equiv i_t - \pi_{t+1|t}^d$ .

## 2.3 The public sector and the rest of the world

The behavior of the central bank consists of minimizing the following loss function:

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left[ \mu^c \pi_{t+\tau}^{c2} + \mu^d \pi_{t+\tau}^{d2} + \lambda y_{t+\tau}^{d2} + \nu (i_{t+\tau} - i_{t+\tau-1})^2 \right], \quad (28)$$

where  $\mu^c$ ,  $\mu^d$ ,  $\lambda$  and  $\nu$  are weights that express the preferences of the central bank for CPI and domestic inflation targets, the output stabilization target, and the instrument smoothing target, respectively<sup>7</sup>.

The rest of the world is exogenous and described by stationary univariate AR(1) processes

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<sup>7</sup>Regarding the motivation for an interest rate smoothing preferences in the Central Bank loss function see, for example, Svensson (2010), Holmsen et al. (2008), and Flamini and Fracasso (2011).

for foreign inflation and income, and a Taylor rule for monetary policy, respectively

$$\pi_{t+1}^* = \gamma_\pi^* \pi_t^* + \varepsilon_{t+1}^*, \quad (29)$$

$$y_{t+1}^* = \gamma_y^* y_t^* + \eta_{t+1}^*, \quad (30)$$

$$i_t^* = f_\pi^* \pi_t^* + f_y^* y_t^* + \zeta_t^*, \quad (31)$$

where the shocks are white noises.

## 2.4 Certainty non-equivalence and model parameter uncertainty

The presence of uncertainty on some structural parameters introduces multiplicative uncertainty in the model. This implies that the certainty-equivalence principle does not hold anymore and the optimal policy in presence of uncertainty differs from the one in presence of certainty. To model multiplicative uncertainty and compute the equilibrium the current work follows the Markov Jump-Linear-Quadratic approach developed by Svensson and Williams (2007). Accordingly, the behaviour of the private sector described by equations (16, 19, 22, 24, 27-31) is conveniently rewritten in State-space form to obtain the law of motion of the economy. Then, the central bank problem is to find the expected interest rate path that minimizes its loss given the law of motion of the economy, that is

$$\text{Min}_{\{i_{t+\tau|t}\}_{\tau=0}^\infty} E_t \sum_{\tau=0}^\infty \beta^\tau Y'_{t+\tau} K Y_{t+\tau}$$

subject to

$$\begin{bmatrix} X_{t+1} \\ x_{t+1|t} \end{bmatrix} = \begin{bmatrix} A_{11,t+1} & A_{12,t+1} \\ A_{21,t} & A_{22,t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_{1,t+1} \\ B_{2,t} \end{bmatrix} i_t + \begin{bmatrix} B_{1,t+1}^1 \\ B_{2,t}^1 \end{bmatrix} i_{t+1|t} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix},$$

$$Y_t \equiv C_{Z,t} \begin{bmatrix} X_t \\ x_t \end{bmatrix} + C_{i,t} i_t,$$

where the target variables, the predetermined variables, and the forward looking variables are, respectively

$$\begin{aligned} Y_t &= \left( \pi_t^c, \pi_t^d, y_t^d, i_t - i_{t-1} \right)', \\ X_t &= \left( \pi_t^d, \pi_{t+1|t}^d, \pi_{t-1}^i, \pi_t^*, y_t^d, y_t^i, y_t^*, i_t^*, y_t^{d,n}, y_t^{i,n}, i_{t-1}, q_{t-1}, q_{t-1}^i, v_t \right)', \\ x_t &= \left( \pi_t^i, q_t^i, \rho_t, \pi_{t+2|t}^d \right)', \end{aligned}$$

and where  $K$  captures the central bank's preferences, a diagonal matrix with the diagonal  $(\mu^c, \mu^d, \lambda, \nu)$  and off-diagonal elements equal to zero. Following the Markov Jump-Linear-Quadratic approach developed by Svensson and Williams (2007) I assume that the matrices

$$A_{11,t}, A_{12,t}, B_{1,t}, B_{1,t}^1, A_{21,t}, A_{22,t}, B_{2,t}, B_{2,t}^1, C_{Z,t}, C_{i,t}, \quad (32)$$

are random, each free to take  $n_j$  different values in period  $t$  corresponding to the  $n_j$  modes indexed by  $j_t \in \{1, 2, \dots, n\}$ . This means that, for example,  $A_{11,t} = A_{11,j_t}$ . The mode  $j_t$  is then assumed to follow a Markov process with constant and equal transition probabilities

$$P_{jk} \equiv \Pr \{j_{t+1} = k | j_t = j\} = \frac{1}{n}, \quad j, k \in \{1, 2, \dots, n\}. \quad (33)$$

Furthermore, model parameter uncertainty and shocks to the economy are assumed to be independent so that modes  $j_t$  and innovations  $\varepsilon_t$  are independently distributed. Finally, the central bank is assumed not to know how the structural parameters co-move together, should they be dependent. So in any period the realization of each parameter is independent of the realizations of the other parameters. As to the central bank knowledge before choosing the instrument-plan  $\{i_{t+\tau|t}\}_{\tau=0}^{\infty}$  at the beginning of period  $t$ , the information set consists of the probability distribution of  $\varepsilon_t$ , the transition matrix  $[P_{jk}]$ , the  $n_j$  different values that each of the matrices can take in any mode, and finally the realizations of  $X_t, j_t, \varepsilon_t, X_{t-1}, j_{t-1}, \varepsilon_{t-1}, x_{t-1}, \dots$

Given (33), the unique stationary distribution of the modes associated with the Markov transition matrix  $[P_{jk}]$  is a uniform distribution. This implies that the transition probabilities described by (33) capture the case of generalized modes uncertainty in which modes are serially i.i.d.. The motivation to consider this case lies in the interest of studying optimal monetary policy when the central bank only knows a band for each uncertain deep parameter and considers

any realization as equally likely. For example, if there is only one uncertain parameter, say  $\xi$ , a benchmark value is chosen,  $\bar{\xi}$ , and the lower and upper bound of the support of the distribution are set equal to  $\bar{\xi} - x\bar{\xi}$  and  $\bar{\xi} + x\bar{\xi}$  respectively, where the coefficient  $x$  modules the variance of the distribution and therefore the amount of uncertainty. Turning to the number of modes, letting  $m$  be the number of uncertain parameters and  $d$  be the number of values that each parameter can take in any period, then the number of modes is  $n = d^m$ . In this work  $d = 5$  and  $m$  can be either 1 or 2 or 5 depending on the uncertainty cases described below.

With respect to the solution of the optimal monetary policy problem, I address the impossibility to proceed analytically and the presence of forward looking variables with the numerical methods developed by Svensson and Williams (2007). Adopting this approach, I find the equilibrium in the presence of multiplicative uncertainty under commitment in a timeless perspective (see Woodford 2003 and Svensson and Woodford 2005)<sup>8</sup>. Numerical methods, in turn, require a calibration which is presented in the following section.

## 2.5 Calibration

Two groups of parameters need to be calibrated to solve the model. The first consists of the parameters that are assumed to be known with certainty, while the second the benchmark values for the uncertain parameters.

The choice of the parameters assumed to be known with certainty follows Svensson (2000) as the current model is similar in structure to the Svensson's one. These parameters, with respect to the domestic economy, are the output elasticity of the marginal input requirement function,  $\omega = 0.8$ ; the elasticity of substitution between varieties of the same type of good  $\vartheta = 1.25$ ; the intertemporal elasticity of substitution,  $\sigma = 0.5$ ; the share of import good in the composite input to produce the domestic good,  $\mu = 0.1$ ; the share of import goods in domestic consumption,  $w = 0.3$ . With respect to the foreign sector, the elasticity of substitution between domestic and import goods for foreign consumers is  $\theta^* = 2$ ; the share of the domestic good in foreign consumption is  $w^* = 0.15$ ; the income elasticity of foreign real consumption is  $\bar{\beta}_y^* = 0.9$ ; and the coefficients for the foreign Taylor rule are  $f_{\pi^*} = 1.5$ , and  $f_{y^*} = 0.5$ . Finally, the exogenous cost push and demand shocks have variances  $\sigma_{\pi}^2 = \sigma_y^2 = 1$ ; the natural output shocks have variances  $\sigma_{y^{d,n}}^2 = \sigma_{y^{i,n}}^2 = 0.5$  and AR(1)-parameter  $\gamma_y^{d,n} = \gamma_y^{i,n} = 0.96$ , and finally the risk premium, foreign inflation and output have AR(1) process-parameter  $\gamma_{y^*} = \gamma_{\pi^*} = \gamma_v = 0.8$  and variances

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<sup>8</sup>The implementation of these methods, which are used to obtain the figures and tables shown below is carried out coding in Matlab.

$\sigma_\nu^2 = \sigma_{\pi^*}^2 = \sigma_{y^*}^2 = 0.5$ . As to the central bank preferences, the weights in the loss function under DIT and CPI IT are, respectively,  $\mu^d = 1$ ,  $\mu^c = 0$  and  $\lambda = 0.5$ , and  $\mu^d = 0$ ,  $\mu^c = 1$  and  $\lambda = 0.5$ .

The benchmark values of the uncertain parameters follow Banerjee and Batini (2003) as to the measure of habit formation in the utility function,  $\bar{\iota} = 0.8$  and Smets and Wouters (2005) as to the degree of indexation to the previous period inflation rate,  $\bar{\zeta} = 0.66$ . The probability on not optimally updating the price in the current period in the domestic and import sector,  $\bar{\alpha}$ , and  $\bar{\alpha}^i$ , are set equal to 0.5 following Svensson (2000) and Flamini (2007), respectively. Finally, the value of the share of domestic good in the composite input to supply the import good,  $\bar{\mu}^i$ , is set to 0.35 consistently with Flamini (2007) and such that the lower and upper bound of the support of the  $\mu^i$  distribution are realistic for the uncertainty level considered in the analysis; specifically the lower and upper bounds are 0.245 and 0.405.

### 2.5.1 Robustness check

The current model is also similar in spirit to the Leitemo and Söderström (2005) model. Although the latter is not microfounded, its parametrization for the exogenous disturbances provides a valid alternative to check for the robustness of the results. In the Leitemo and Söderström model, the cost-push shock and the demand shock are AR(1) processes and their AR(1)-coefficients,  $\gamma_\pi$  and  $\gamma_y$ , are set equal to 0.3 (this is a difference with the previous calibration where the AR(1)-coefficients for these two shocks are implicitly set equal to zero). The variances for these shocks are  $\sigma_y^2 = 0.656$  and  $\sigma_\pi^2 = 0.389$ , while the variance for the shocks to the risk premium, foreign inflation, and foreign output gap are  $\sigma_\nu^2 = 0.844$ ,  $\sigma_{\pi^*}^2 = 0.022$ , and  $\sigma_{y^*}^2 = 0.083$ , respectively<sup>9</sup>. For the risk premium AR(1)-coefficient  $\gamma_\nu$ , Leitemo and Söderström considers the interval  $[0, 1]$ . In the current analysis, having to choose one value,  $\gamma_\nu$  is set equal to 0.5.

To recap, all the parameters known with certainty and associated with the Svensson (2000) and the Leitemo and Söderström (2005) calibrations are reported, respectively in Panels a and b of Table 1, while the benchmark values of the uncertain parameters are reported in Table 2.

## 3 Forecasts accuracy under DIT and CPI IT

Model uncertainty poses a major challenge to real world monetary policy. In this work, the consideration of model parameter uncertainty is what allows moving from *mean forecast target-*

<sup>9</sup>Leitemo and Roisland (2002) find these variances with a structural VAR on the Norwegian economy.

ing to *distribution forecast targeting*. The latter means that, given a specific policy, e.g. DIT or CPI IT, and given an exogenous disturbance, the solution of the optimization problem implies a correspondence for which each point in time corresponds to a distribution for each macrovariables. This information richness is lost with mean forecast targeting where, due to the certainty equivalence principle, the optimal policy response to an exogenous shock implies a function for which each point in time corresponds exactly to one value of the macrovariables. Thus the relevance of accounting for model parameter uncertainty lies in shedding light on the expected volatility of the macrovariables at any current and future point in time, which is an important aspect of the economic outlook associated with different monetary policies and contingencies.

### 3.1 Distribution forecasts to a cost-push shock in presence of general uncertainty

The analysis starts with the unconditional distribution forecasts of the impulse responses to a (one standard deviation) cost-push shock reported in Figures 1-2. The distribution forecasts are generated assuming *general* uncertainty, which encompasses uncertainty on the pass-through,  $(\mu_j^i, \alpha_j^i)$ , on the persistence in the private sector's behaviour,  $(\iota_j, \zeta_j)$ , and on the slope of the domestic AS,  $(\alpha_j)$ . In each figure, the first and second column report the distribution forecasts of the main macrovariables under the optimal policies of domestic and CPI IT respectively. Assuming an uncertainty level of 30% on all the uncertain parameters, these figures have been generated by drawing an initial mode of the Markov chain from its stationary distribution, simulating the chain for a sequence of periods forward, and then repeating this procedure for 1000 simulations runs<sup>10</sup>. Thus these figures display mean (dashed line), and quantiles (grey bands), of the empirical distribution. In particular, the dark, medium and light grey band show the 30%, 60%, and 90% probability bands, respectively. Figures 1-2 consider, respectively, high and low central bank preferences for smoothing the interest rate path<sup>11</sup>. High attention on smoothing the interest rate implies a mild monetary policy where there is almost no attempt to buffer the shock. This case is interesting as starts to reveal the impact of model parameter uncertainty and alternative inflation indexes on the distribution forecasts; it thus provides a benchmark. In the latter case, low preferences for interest rate smoothing, the monetary policy is more realistic and the different impact of model parameter uncertainty on the distribution

<sup>10</sup>The results presented in this and the next sections are robust to smaller and larger uncertainty levels.

<sup>11</sup>Specifically, the interest rate smoothing preferences parameter,  $\nu$ , in the loss function (28), is 0.05 in Figure 1 and 0.002 in Figure 2.

forecasts linked to alternative target inflation indexes is fully revealed.

Figure 1 features a high preference for interest rate smoothing. Here visual inspection shows that the volatility of the macrovariables distribution tends to be higher under CPI IT. In Figure 2, switching to a low preference for interest rate smoothing, and therefore to a more aggressive policy, the previous result is strongly amplified: DIT implies much less volatility of the projections of the economy, in particular of the interest rates, and a surprisingly better ability to absorb the cost-push shock. It is also interesting to note that CPI inflation,  $\pi^c$ , does not seem to be less volatile under CPI IT. As we would have expected, under CPI IT the optimal monetary policy attempts to absorb the cost-push shock using the exchange rate. This is reflected in the initial decrease of import inflation,  $\pi^i$ , shown in the sixth row, second column.

Summing up, these findings first suggest that buffering a cost-push shock under DIT leads to less volatility in the distribution forecasts than under CPI IT. Second, if the central bank is called to set a less smooth interest rate path, i.e. a more aggressive policy, then CPI IT leads to much more expected volatility in the economic outlook than DIT.

### 3.2 Measuring the volatility of $i$ and $y^d$ in presence of a cost-push shock

On the basis of the previous analysis with high and low interest smoothing preferences, a natural question to ask is whether the volatility of the macrovariables is monotonous in the preferences for smoothing. This is relevant given the uncertainty on the smoothing preferences of the central bank and, more in general, the time varying degree of activism in monetary policy possibly related to central bank judgment. To address this question, Figure 3 focuses on the cost-push shock case and presents the *standard deviation* of the distribution forecasts of the nominal interest rate and the domestic output-gap for the periods considered above and for interest rate smoothing values in the set  $V = \{0.002, 0.005, \dots, 0.04\}$ <sup>12</sup>. Explaining this figure, each sub plot reports two surfaces that describe the standard deviation of the distribution forecasts under CPI and DIT. The first and the second row refer to the interest rate and the output gap, respectively, while the columns refer to four uncertainty cases, specifically uncertainty (i) on the pass-through, (ii) on the persistence of the behaviour of households and firms, (iii) on the degree of price flexibility in the domestic sector (AS slope uncertainty), and (iv) on all the previous sources, i.e. general uncertainty.

A first result considering the interest rate (first row) is that either the CPI IT surface is

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<sup>12</sup>Section 3.4. and 3.5 will extend the analysis to other macrovariables and shocks.

always above the DIT surface (in the uncertainty on the pass-through, on the persistence in the behaviour of households and firms, and general uncertainty cases, first, second, and forth column respectively), or the two surfaces tend to overlap with the DIT one slightly above the CPI one for small preferences on interest rate smoothing (in the cases of uncertainty on the slope of the Phillips curve in the domestic sector, third column). This shows that under the pass-through, persistence, and general uncertainty cases the CPI IT policy results systematically in a larger standard deviation for the distribution forecast of the interest rate than DIT. Instead, when we consider the case of uncertainty on the degree of price flexibility in the domestic sector, the standard deviation associated with DIT tends to be higher than the one associated with CPI IT. Moving to the second row describing the variability of the distribution forecast of the output gap in the domestic sector we obtain similar results.

Second, the volatility of the distribution forecasts of the interest rate and the output gap tend to be monotonically increasing in the preference for not smoothing the interest rate. Yet, it is interesting to note that, decreasing interest rate smoothing, the volatility under CPI IT tends to increase more than under DIT.

These findings are relevant as they generalize to a broad set of interest rate smoothing preferences the previous findings reported in Figures 1-2: DIT leads to less variability of the distribution forecasts of the interest rate and of the output gap in the presence of a cost-push shock, and it is less sensitive to interest rate smoothing.

In order to quantitatively compare the volatility of the distribution forecasts associated with the two policies it is informative to compute the ratio of the means (along all the smoothing preferences values and the periods considered) of the standard deviations in the two policy cases, i.e.

$$R^\sigma \equiv \frac{\text{mean} |_{\nu,t} \text{std}_{\nu,t}^c(\text{variable})}{\text{mean} |_{\nu,t} \text{std}_{\nu,t}^d(\text{variable})},$$

where  $\text{std}_{\nu,t}^h(\text{variable})$ ,  $h = c, d$ , denote the standard deviation of the distribution forecast of the considered variable for period  $t$ , and smoothing preferences value  $\nu$ , and  $c$  and  $d$  denote CPI and DIT, respectively. Table 3 considers the nominal interest rate and the domestic output gap and presents the statistics  $R^\sigma$  for various uncertainty types.

INSERT TABLE 3 HERE

This analysis shows that for the nominal interest rate, in almost all uncertainty cases, DIT

dominates CPI IT. Furthermore, when we focus on the more representative case of general uncertainty, which includes all the previous cases, the mean of the standard deviation under CPI IT is 2.79 times larger than under DIT. Considering the output gap, DIT dominates CPI IT in all the uncertainty cases except the one of uncertainty on the slope of the aggregate supply where they tend to be equivalent. In the general uncertainty case the average variability of the distribution forecast for the output gap with the CPI policy is 1.48 times larger than with the other policy.

### 3.3 Targeting policies and forecast accuracy: the overall economic outlook

Do the earlier results associated with the  $R^\sigma$  statistics hold for the other macrovariables and external disturbances? This section shows that earlier findings tend to hold to a remarkable extent in a more general setting. Considering also CPI and domestic inflation,  $\pi^c$  and  $\pi^d$  respectively, the short term real interest rate,  $r$ , and the real exchange rate,  $q$  along with the additional (one standard deviation) shocks to the aggregate demand, the foreign interest rate, the natural output, the risk premium, and the foreign output, Tables 4-5 report the  $R^\sigma$  ratio for the *general uncertainty* case.

INSERT TABLES 4-5 HERE

To discuss the results it is useful to define three levels of dominance in terms of intervals for the ratios  $R^\sigma$ . These levels of dominance are

$$\begin{aligned}
 \text{Strong Dominance} & \iff 0 < R^\sigma \leq 0.5 \text{ or } R^\sigma \geq 2 \\
 \text{Dominance} & \iff 0.5 < R^\sigma < 0.9 \text{ or } 1.\bar{1} < R^\sigma < 2 \\
 \text{Weak Dominance} & \iff 0.9 \leq R^\sigma \leq 1.\bar{1}
 \end{aligned}$$

Describing these intervals, the “Strong Dominance” case is the case in which one policy leads to a volatility at least twice as large as the other. The “Weak Dominance” case is the case in which one policy leads to a volatility at the most nine tenth as large as the other. In turn, the Strong and Weak Dominance intervals delimit the intervals in between which define the “Dominance” case. While the distinction between Strong Dominance and Dominance cases aims to capture remarkable differences in the intensity of dominance, the motivation for introducing Weak Dominance intervals is to identify and filter out close calls, i.e. similar performances potentially difficult to make a decision about.

Turning to the results, Table 4 describes the performance of the two policies under the Svensson (2000) calibration. Abstracting from the *weak dominance* cases, DIT is *strongly dominant* or *dominant* in 44.4% of the cases, while it is dominated in 27.7% of the cases<sup>13</sup>. Interestingly, DIT strongly dominates in approximately one fifth of the cases, yet it is never strongly dominated. Checking for the robustness of these results, the analysis based on the Leitemo and Söderström (2005) calibration corroborates the previous findings. Indeed, results in Table 5 show that DIT is strongly dominant or dominant in the 63.8% while it is dominated in the 16.6% of the cases.

It is worth noting that the cases in which DIT is dominated tend to pertain to CPI inflation, as we would expect, and also to the real exchange rate. As to the former, except for the cost-push shock, both the distribution forecasts of domestic and CPI inflation are not very sensitive to exogenous disturbances. Thus the two policies tend to be similar in their ability to stabilize inflation even if each one is better at stabilizing its own measure of inflation<sup>14</sup>. As to the latter, the real exchange rate, with a demand, natural output, risk premium, and foreign output shock, CPI IT performs better as is shown in Table 4-5. This is due to the fact that it aims to stabilize both domestic and import inflation, which determine the real exchange rate.

Shocks to the risk premium, foreign interest rate and foreign output gap deserve a final comment. In these cases the shocks impact on the nominal exchange rate via the uncovered interest parity. Then, if the central bank does not react, the shock propagates to CPI inflation. Thus with CPI IT the central bank has to respond to these shocks. Yet, the central bank may not be willing to react to shocks that affect the nominal exchange rate. Leitemo and Söderström (2005) maintain that it should not. Their argument is that there is uncertainty about how the exchange rate is determined and the effect of exchange rate movements on the economy. This implies that rules with the exchange rate are more sensitive to model uncertainty. Thus a monetary policy developed in the context of an exchange rate model could perform poorly if that model is incorrect. Empirical evidence in this respect is not conclusive. Lubik and Schorfheide (2007) find that Australia and New Zealand did not react to movements in the exchange rate while Canada and the UK did.

Describing the mechanism that generates these results, two factors stand out: more policy activism under CPI IT than under DIT and the presence of model parameter uncertainty.

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<sup>13</sup>DIT is strongly dominant in 8 cases, dominant in 8 cases, weakly dominant in 4 cases, weakly dominated in 6 cases, dominated in 10 cases, and strongly dominated in 0 cases.

<sup>14</sup>The impulse response distribution forecasts for the complete set of shocks are available upon request.

The first factor is shown in Figures 4-5 computed assuming no model parameter uncertainty. These figures displays the impulse response function of the nominal interest rate to a cost-push shock under the two alternative policies for high and low smoothing preferences, Figure 4 and 5 respectively. Measuring monetary policy activism by the volatility (in terms of std) of the impulse response function around its long run value, under CPI IT this volatility is 1.3 times larger than under DIT when  $\nu = 0.05$ , and 4.53 times larger when  $\nu = 0.002$ .

More policy activism under CPI IT than under DIT is due to i. different lags in the transmission of the policy action to CPI and domestic inflation, and ii. to a larger exposure of CPI inflation to foreign shocks. Different lags arise as the pricing decisions for domestic firms embed not only retailing decisions but production decisions too, and therefore are more subject to information delays. It follows a longer lag for policy action to affect domestic inflation than CPI inflation via the output gap. This is the policy transmission that occurs through the aggregate demand channel and the switching demand exchange rate channel. It follows also that shocks to the exchange rate and the price of the foreign goods in foreign currency affect domestic inflation with a lag via  $q_t$  in the AS for the domestic sector, while they affect directly import inflation via  $q_t^i$  in the AS for the import sector<sup>15</sup>.

Furthermore, more policy activism depends on a larger exposure of CPI inflation to foreign shocks. Indeed, via the uncovered interest parity, the latter cause exchange rate volatility exerting a stronger impact on CPI inflation than domestic inflation because import sector inputs are more intensive in foreign goods than domestic sector inputs. As a result, under CPI IT the central bank is more solicited to intervene in order to prevent exchange rate volatility from leading to too much CPI inflation volatility. Hence, CPI IT implies a more pronounced trade-off between CPI inflation and interest rate volatility.

What happens when more policy activism is associated with the consideration of model parameter uncertainty in the design of the optimal monetary policy? When model uncertainty is taken into account we move from one expected path for the interest rate (Figures 4-5) to a set of expected paths, which form the distribution forecast for the interest rate (third row in Figures 1-2). At this point, the degree of policy activism expands the width of the distribution forecast. Indeed, the larger the initial monetary policy stimulus, the more the uncertainty on the private sector behavior can lead to future changes in the policy.

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<sup>15</sup>The impact of the exchange rate on the domestic price of the foreign good is amply documented in the literature and usually referred to as the Direct Exchange Rate channel.

Finally, a wider distribution forecast for the interest rate results in wider distribution forecasts for most of the other macrovariables, which is the result shown in Figure 1-2 and reported, more generally, in Tables 4-5.

## 4 Conclusions

When the central bank designs the optimal monetary policy considering the uncertainty on the parameters of the model economy, exogenous shocks generate a continuum of possible expected paths for the macrovariables which constitute distribution forecasts. Simulating the response of the economy to exogenous shocks and taking the volatility of the distribution forecasts as a measure of forecast accuracy, this paper argues that the choice of the inflation targeting policy can significantly affect forecast accuracy. Specifically, among two alternative targeting policies for a small open-economy: CPI IT and DIT, the latter stands out as the policy associated with less volatility in the response of the economy to the shock and therefore featuring more forecast accuracy.

The paper also shows that there is a trade-off between forecast accuracy and interest rate smoothing, and that this trade-off is more severe under CPI IT. Thus, noting that interest rate smoothing and policy aggressiveness are inversely related, this result matters as if the central bank needs to buffer more vigorously a shock, then DIT turns out to compromise forecast accuracy less than CPI IT.

Forecast accuracy in macroeconomics has been so far an empirical field based on statistical techniques for extrapolating time series. *Via* a new theoretical route, this work shows that forecast accuracy can remarkably depend on the choice of the type of inflation targeting monetary policy.

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TABLE 1 Parameters known with certainty

Panel a (Svensson 2000)						Panel b (Leitemo and Söderström (2005))	
$\omega$	0.8	$\theta^*$	2	$\sigma_\pi^2, \sigma_y^2$	1	$\gamma_\pi, \gamma_y$	0.3
$\vartheta$	1.25	$w^*$	0.15	$\sigma_{y^{d,n}}^2, \sigma_{y^{i,n}}^2$	0.5	$\sigma_y^2$	0.656
$\sigma$	0.5	$\bar{\beta}_y^*$	0.9	$\gamma_y^{d,n}, \gamma_y^{i,n}$	0.96	$\sigma_\pi^2$	0.389
$\mu$	0.1	$f_{\pi^*}$	1.5	$\gamma_{y^*}, \gamma_{\pi^*}, \gamma_v$	0.8	$\sigma_v^2$	0.844
$w$	0.3	$f_{y^*}$	0.5	$\sigma_v^2, \sigma_{\pi^*}^2, \sigma_{y^*}^2$	0.5	$\sigma_{\pi^*}^2$	0.022
						$\sigma_{y^*}^2$	0.083
						$\gamma_v$	0.5
CPI IT	$\mu^d = 0, \mu^c = 1, \lambda = 0.5$						
DIT	$\mu^d = 1, \mu^c = 0, \lambda = 0.5$						

TABLE 2 Benchmark values of the uncertain parameters

$\bar{i}$	0.8	Banerjee and Batini (2003)
$\bar{\zeta}$	0.66	Smets and Wouters (2005)
$\bar{\alpha}, \bar{\alpha}^i$	0.5	Svensson (2000)
$\bar{\mu}^i$	0.35	Flamini (2007)

TABLE 3  $R^\sigma$  for various uncertainty type. Shock: cost-push. First calibration.

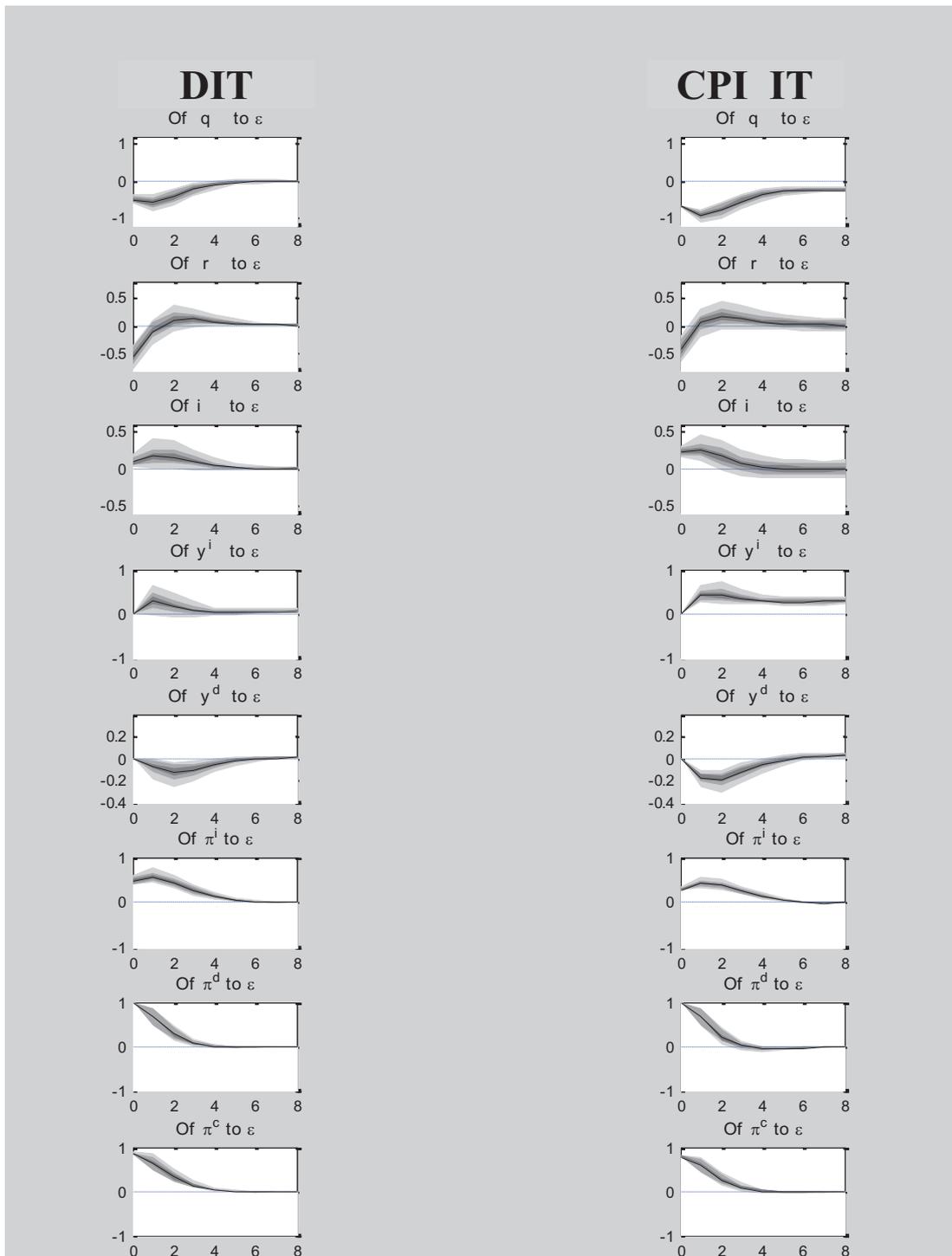
Uncertainty type	$i$	$y^d$
Pass-through	3.68	2.27
Persistence private sector behavior	1.16	1.11
Domestic AS slope	0.91	1.01
General	2.79	1.48

TABLE 4  $R^\sigma$  for various shocks and variables under general uncertainty. First calibration.

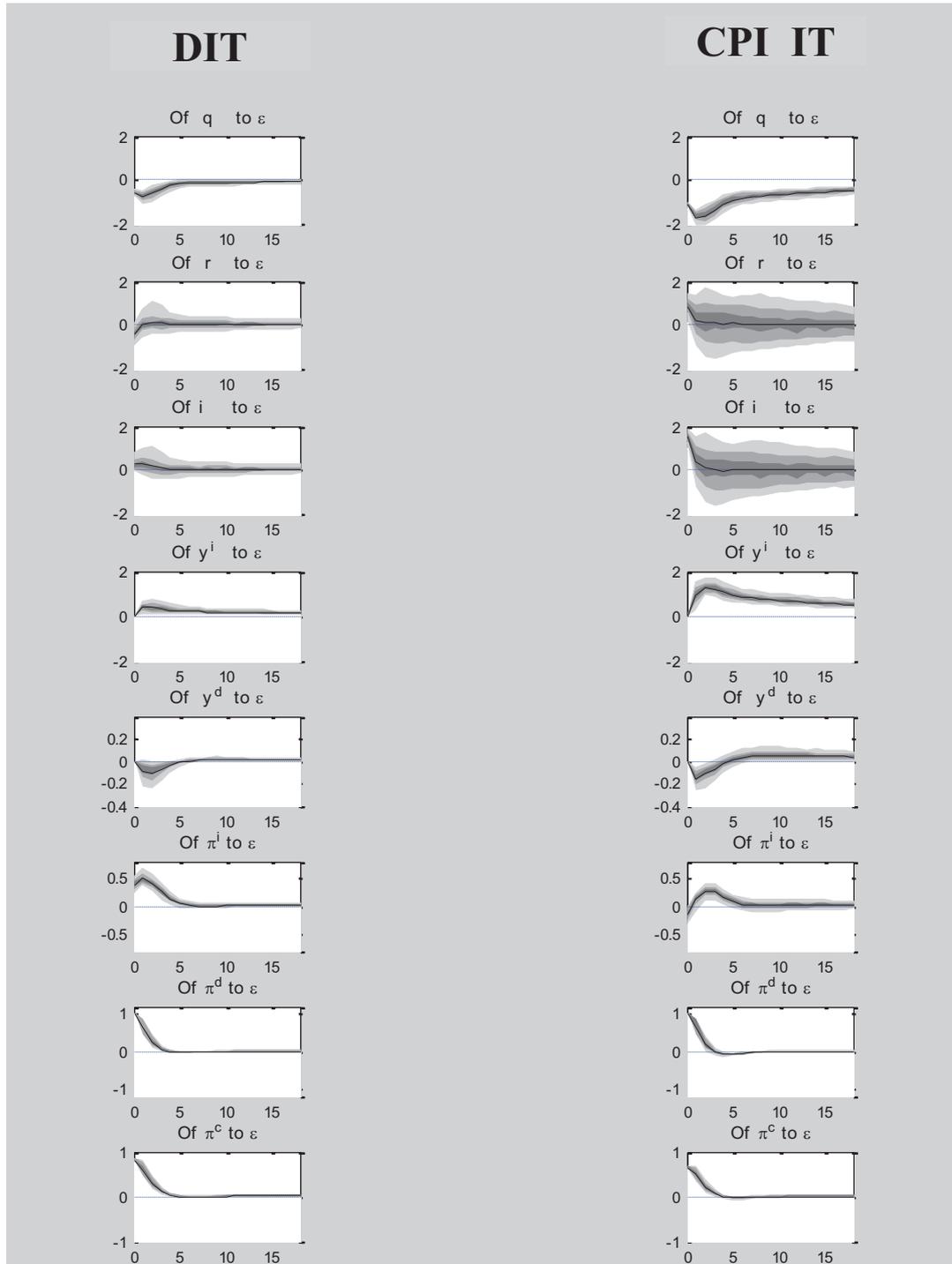
Shock	$\pi^c$	$\pi^d$	$y^d$	$i$	$r$	$q$
Cost-push	1.08	1.15	1.48	2.79	2.62	1.44
Demand	0.89	1.16	0.95	1.05	1.05	0.82
Foreign interest rate	0.77	1.32	1.18	2.91	2.77	1.01
Natural output	0.87	1.11	0.98	0.97	0.97	0.75
Risk premium	0.71	0.90	0.82	2.09	2.05	0.77
Foreign output	0.76	1.16	0.94	2.22	2.26	0.88

TABLE 5  $R^\sigma$  for various shocks and variables under general uncertainty. Second calibration.

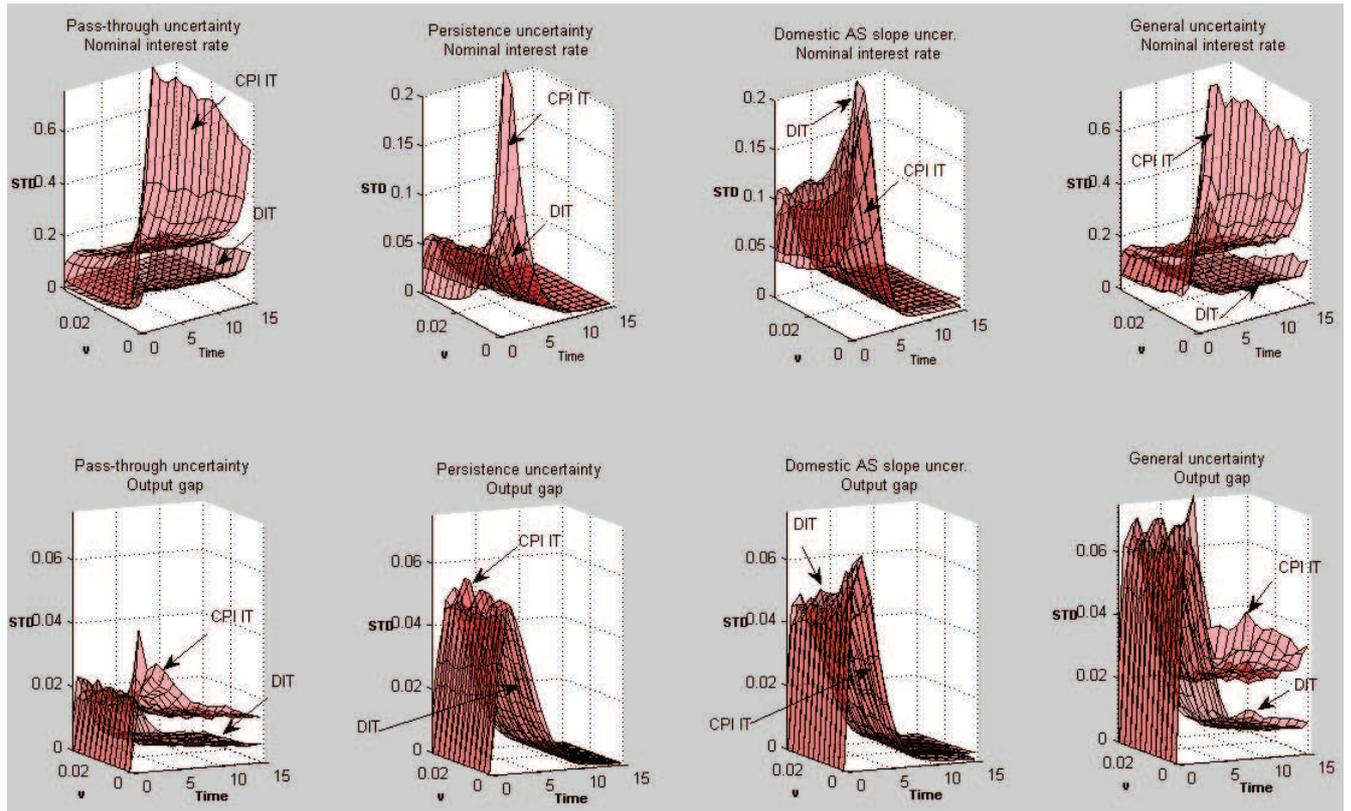
Shock	$\pi^c$	$\pi^d$	$y^d$	$i$	$r$	$q$
Cost-push	1.05	1.13	1.23	1.79	1.22	2.04
Demand	0.86	1.16	0.91	0.94	1.17	1.01
Foreign interest rate	0.76	1.31	1.19	2.23	1.35	2.91
Natural output	0.87	1.12	0.99	0.89	1.12	0.95
Risk premium	0.74	1.33	1.19	2.67	1.38	3.35
Foreign output	0.77	1.15	0.94	1.75	1.19	2.22



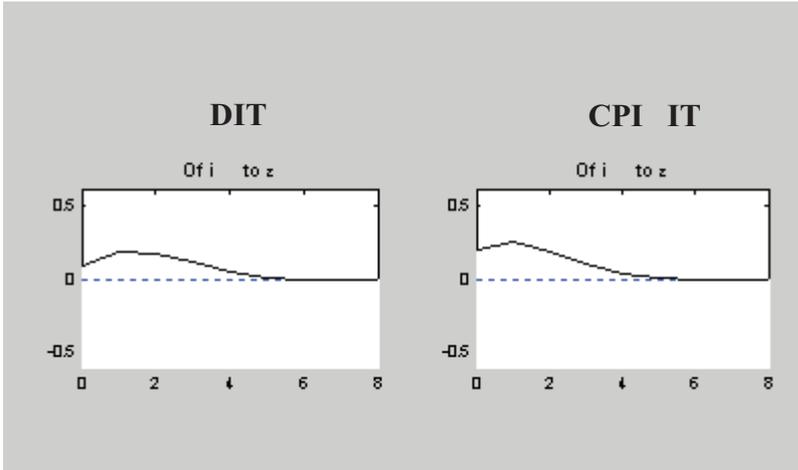
**Figure 1:** Unconditional distribution forecasts of the impulse responses to a cost-push shock in the *general uncertainty* case and for high smoothing preferences, i.e.  $\nu = 0.05$ . First and second column report, respectively, the distribution forecasts under the DIT and CPI IT policies. Solid lines: Mean responses. Dark/medium/light grey bands: 30/60/90% probability bands. First calibration.



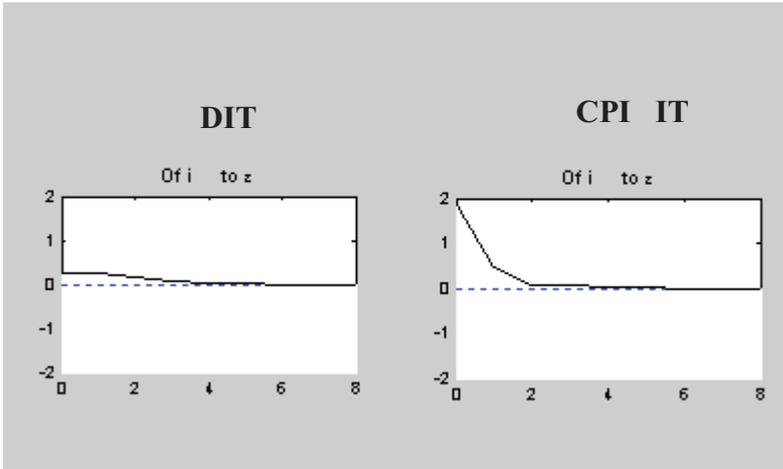
**Figure 2:** Unconditional distribution forecasts of the impulse responses to a cost-push shock in the *general* uncertainty case and for low smoothing preferences, i.e.  $\nu = 0.002$ . First and second column report, respectively, the distribution forecasts under the DIT and CPI IT policies. Solid lines: Mean responses. Dark/medium/light grey bands: 30/60/90% probability bands. First calibration.



**Figure 3:** STD of the impulse response distribution to a cost-push shock under DIT and CPI IT for  $v \in \{0.002, 0.005, \dots, 0.04\}$  and  $t \in \{0, 1, \dots, 15\}$ . Variables:  $i$  and  $y^d$ , first and second row respectively. Uncertainty cases: pass-through, persistence in the behaviour of the private sector, slope of the domestic AS, and general, first, second, third and fourth column respectively. First calibration.



**Figure 4:** Impulse response of the nominal interest rate to a cost push-shock assuming no model uncertainty and for high smoothing preferences, i.e.  $\nu = 0.05$ .



**Figure 5:** Impulse response of the nominal interest rate to a cost push-shock assuming no model uncertainty and for low smoothing preferences, i.e.  $\nu = 0.002$ .