UNIVERSITÀ DEGLI STUDI DI MILANO

UNIVERSITÀ DI PAVIA

Ph.D. in ECONOMICS – Universities of Milan and Pavia

Static and Dynamic Optimization

Academic year 2026-27 –First Term

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Office hours: by appointment

Course description: The course introduces students to static and dynamic optimization. The analysis focuses on the development of some mathematical tools that are fundamental for the study of advanced models in economic theory

Learning objectives: The course will offer an organic overview of some mathematical tools used in economic theory. The students will learn how to use some mathematical notions and results in economic models under appropriate assumptions and formalizations.

Learning outcomes: The students will learn how to solve some static and dynamic optimization problems using necessary and sufficient optimality conditions in a framework where the role of the assumptions is carefully considered.

Course prerequisites: Some basic notions on differential calculus and linear algebra are considered as prerequisites for the course.

Course organization: 20 hours of lectures, to be held in Via San Felice al Monastero, Pavia

Course Assessment: The assessment is based on a written exam with exercises.

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COURSE OUTLINE

1. Linear algebra and quadratic forms. Eigenvalues and eigenvectors, quadratic forms and their sign, constrained quadratic forms

2. Differential calculus. Basic calculus on functions of several variables: partial derivatives and gradient, differentials, Taylor's formula, chain rules of differentiation. Concave functions and their properties. Quasiconcave functions. Implicit function theorems and their applications to comparative static analysis.

3. Unconstrained and constrained optimization. Unconstrained optimization problem: optimality conditions. Concave problems. Parametric unconstrained optimization problems. Optimization problem with equality constraints: Lagrangian function and optimality conditions. Concave problems. Parametric equality constrained problems. Optimization problems with inequality constraints: Kuhn-Tucker optimality conditions.

4. Dynamical systems. Differential and difference equations. Systems of differential and difference equations. Equilibrium solutions for dynamical systems and their stability. The linear case: solutions and stability of equilibrium solutions. Nonlinear case: linearization and Liapunov method.

5. Dynamic optimization. Necessary conditions for optimality in the finite horizon case (Pontryagin Maximum Principle). Economical interpretation of the Maximum Principle. Transversality conditions. Sufficient conditions for optimality. Autonomous problems. Multidimensional optimal control problems. The infinite horizon case.

6. Dynamic programming. Dynamic optimization and Bellman's principle. Finite horizon: recursive method. Infinite horizon: Bellman's equation. Characterization of optimal plans. Concave problems and further properties of the value function.

References

Sections 1,2,3:
Simon-Blume, Mathematics for Economists, WWNorton&Co.
Chapters: 14,15,16,17,18,19,21,23,30(30.3,30.4,30.5).
Other references:
Takayama, Mathematical Economics, CUP

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Chiang, Methods of Mathematical Economics, Mc Graw Hill
De la Fuente, Mathematical Methods and Models for Economists, CUP
Beavis-Dobbs, Optimization and Stability Theory for Economic Analysis, CUP

Sections 3,4,5:
Simon C.P. and Blume L.E., Mathematics for Economists, W.W. Norton and Company,
Chapters 24 and 25;
A.C. Chiang, Methods of Mathematical Economics, Mc Graw Hill, Singapore,
Chapters 13,14 and 18
A. C. Chiang, Elements of Dynamic optimization, Mc Graw Hill, Singapore, 2002,
PART 3 Chapters 7, 8 and 9;
Leonard D. and Van Long N., Optimal Control Theory and Static Optimization in Economics, CUP Cambrige, 1992,
Chapters 4,6,7 and 9;
Intrilligator M.D., Mathematical Optimization and Economic Theory, Prentice-Hall, Inc.,
Englewood Cliffs, N.J., 1971,
Chapters 11, 13 and 14;
De La Fuente A., Mathematical Methods and Models for Economists, Cambridge University Press,
Chapter 12;
M.I. Kamien, N,L. Schwartz, Dynamic Optimization, The calculus of variations and optimal Control in Economics and Management, North-Holland,
PART II Sections 1-9 and 20;
N.L. Stokey, R.E. Lucas Jr., Recursive Methods in Economic Dynamics, Harvard University Press,
Chapters 2, 4 and 5.1.